Physics of shower development – summary, questions and problems

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Basic concepts

- mean free path $\lambda = \frac{1}{N\sigma} = \frac{M}{\rho N_A \sigma}$, survival probability $P_s(l) = 1 - P(l) = \exp\left(-\frac{l}{\lambda}\right)$
- Q: why does N_A appear?
- N = atom density = nb atoms/cm^3 = material density (mass per volume g/cm^3) x nb atoms per mol (N_A) / molar mass (kg/mol)
- $\frac{dE}{dx}|_{\text{elec}}$ = electronic stopping power: mean energy loss per unit path length (MeV/cm)
- R: mass stopping power = stopping power / material density (MeV cm^2 g^{-1}) NB error on slide 12!
- critical energy $\frac{dE}{dx}|_{\text{elec}}(E_c) = \frac{dE}{dx}|_{\text{rad}}(E_c)$ electronic and radiative stopping power are equal
- radiation length X_0 = distance over which 1-1/e (=63.2%) of energy lost due to Bremsstrahlung
- nuclear interaction length: $\sigma_{\text{tot}} = \frac{A}{N_A \lambda_{\text{int}}}$, survival probability $P = \exp\left(-z/\lambda_{\text{int}}\right)$

Proton survival on heavy target

- 400 GeV proton beam on SHiP target (54 cm Mo λ=15cm, 71 cm W λ=9cm) plus hadron absorber (500 cm Fe λ=17cm)
  - $4*10^{13}*(e^{-54/15}* e^{-71/9}* e^{-500/17}) \sim 4.6e^{-5}$
- Q: These are nuclear interaction lengths, inversely proportional to the inelastic cross section
- elastic (nuclear or Coulomb) scattering and ionization do not affect proton survival and are neglected

Energy deposition processes

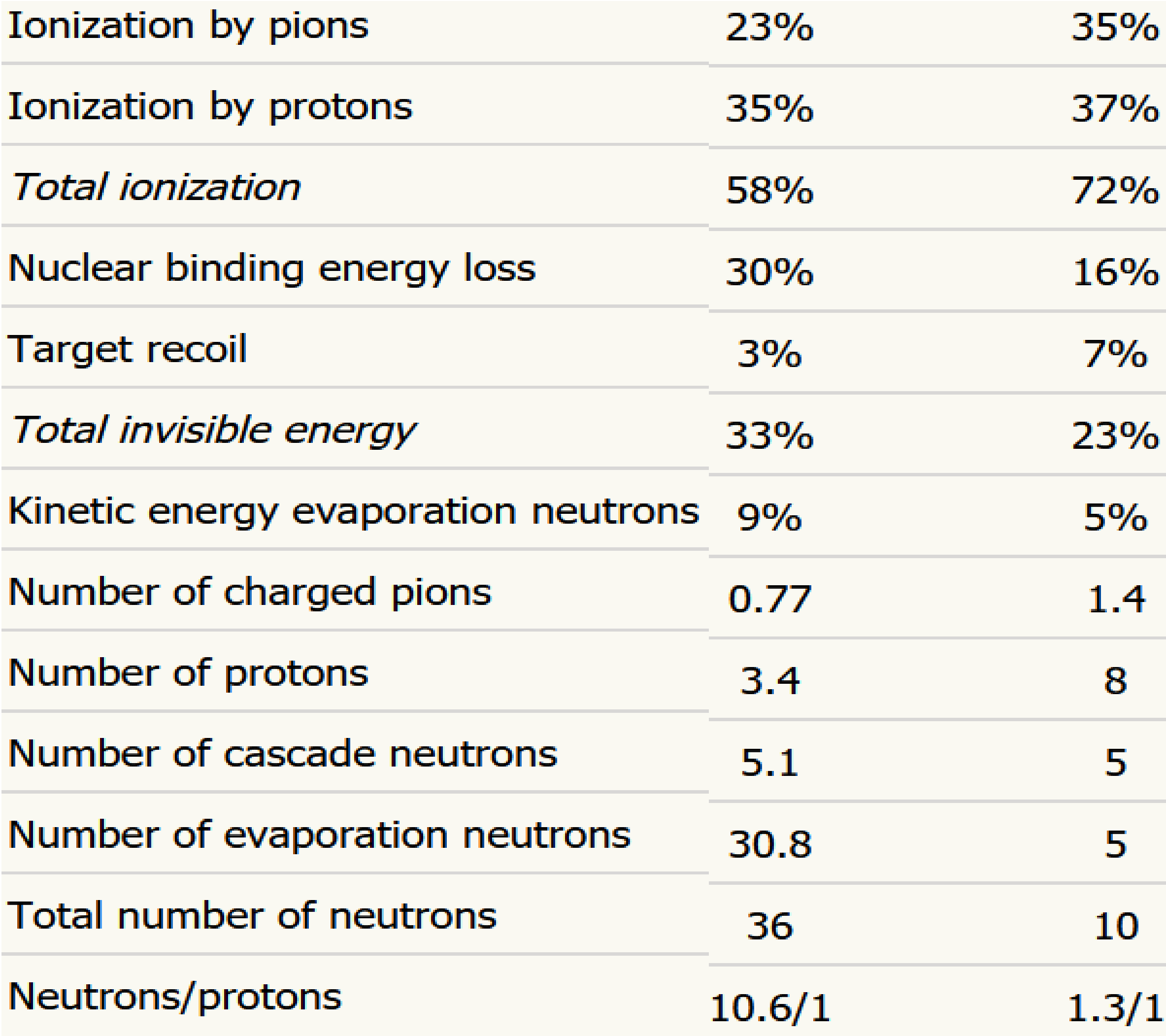
- Charged particles:
  - scattering with nuclei (deflection, E < few MeV)
  - interactions with electrons: ionization (Bethe-Bloch, E_c ~$(m_p/m_e)^2$)
  - Bremsstrahlung ($\frac{dE}{dx}|_{rad} \sim E$, $X_0 \sim A/Z^2\rho$)
- Photons:
  - photo electric effect ($\sigma \sim Z^{4\text{-}5}$ and $\sigma \sim E_\gamma^{-3}$)
  - R: Compton scattering ($\sigma \sim Z$, $\sigma \sim E_\gamma^{-1}$) NB error on slide 15
  - pair production ($\sigma \sim Z^2$, $\sigma \sim \ln(E_\gamma)$)
- Nuclear reactions:
  - spallation: cascade (protons, neutrons), evaporation (neutrons)
  - energy deposition:
    - nuclear binding energy, target recoil energy, ionization by cascade protons, kinetic energy of evaporation neutrons

Electromagnetic and hadronic showers

- Electromagnetic : bremsstrahlung + pair production
  - shower maximum $t_{max} \sim \log(E_0/E_c)$
  - longitudinal development: $L(95\%) = t_{max} + 0.08\,Z + 9.6\,X_0$
  - Moliere radius $\rho_M = \dfrac{\sqrt{4\pi\alpha}\,m_e c^2}{E_c} X_0 \approx \dfrac{21\ \text{MeV}}{E_c} X_0$
  - lateral development: $R(95\%) = 2\,\rho_M$
- Hadronic : nuclear interactions
  - π^0 production → em fraction $f_{em} = 1 - \left(\dfrac{E}{E_0}\right)^{(k-1)}$, $E_0 \sim 1.3$ GeV, $k=0.82$
  - $X_0 \sim \dfrac{A}{\rho Z^2}$, $\lambda_{int} \sim \dfrac{A^{1/3}}{\rho}$
  - $\dfrac{\lambda_{int}}{X_0} \sim A^{4/3}$
  - $\lambda_{int} \gg X_0$, need more material to contain hadronic showers

Energy deposit of non-em component/GeV

- p/n asymmetry in Pb absent for Fe:
    - Coulomb barrier (Pb~12 Mev, Fe~5 Mev)
- nb of nucleons/GeV in Fe (18) smaller than Pb (40):
    - nuclear binding energy (Pb=7.9 MeV, Fe=8.8 MeV). for given energy more (1.1) nucleons released in Pb.
    - p/n asymmetry. Pb (82/208)=39% energy carried by protons, Fe (26/56)=46% → larger (1.2) fraction of energy for excitation in Pb than Fe
    - spallation protons cause a cascade transferring kinetic energy to nucleons. larger (1.6) nucleus (Pb), larger fraction of “contained” energy for evaporation nucleons, increasing the number of nucleons released per unit energy
- 1.1 x 1.2 x 1.6 = 2.1; 40/18=2.2



	Pb	Fe
Ionization by pions	23%	35%
Ionization by protons	35%	37%
Total ionization	58%	72%
Nuclear binding energy loss	30%	16%
Target recoil	3%	7%
Total invisible energy	33%	23%
Kinetic energy evaporation neutrons	9%	5%
Number of charged pions	0.77	1.4
Number of protons	3.4	8
Number of cascade neutrons	5.1	5
Number of evaporation neutrons	30.8	5
Total number of neutrons	36	10
Neutrons/protons	10.6/1	1.3/1

Consequences for calorimetry

- minor role of ionization loss by charged pions in absorption process: ~20 % of non em energy
- major role of ionization loss by protons released from atomic nuclei: ~30-40%
- large fraction of invisible energy: ~30%
- large number of neutrons produced in shower development for high-Z materials

Problems

1. Particle interactions with matter that generate showers
    1. The energy stored in one 7 TeV LHC beam is 362 MJ. How much energy will be released in the SHiP target per spill? Assume:
        - Beam intensity: $4*10^{13}$ protons per spill
        - Beam energy: 400 GeV
    2. What is the probability that a multi GeV photon interacts in 1 cm of lead? The same for iron? How much energy will an electron lose? Calculate using the figure on slide 17, compare your answer with slide 19.
    3. How do 10 MeV photons interact in aluminium?
    4. Calculate the effective radiation length and Molière radius of the SHiP target, which consists of 54.1 cm of Mo, 91.5 cm of W, 5.4 cm of Ta, 8.5 cm of H_2O.
2. Electromagnetic showers
    1. Determine the number of X_0 required to contain 99% of 10 GeV electron showers in lead, copper and aluminium. Compare this with 99% containment.
    2. How much extra X_0 are required to contain showers of 20 GeV photons?
    3. How much lead is required to absorb 90% of the energy contained by 100 GeV electrons?
    4. What is the survival probability of 100 GeV muons after 300m of iron?
    5. Does an electromagnetic shower contain more electrons or positrons? Why?
    6. Compare the Molière radius of copper to that of lead and discuss the impact on shower width in both materials.
3. Hadronic showers
    1. What is the major difference between em and hadronic showers?
    2. Calculate the *punch through* probability for 100 GeV protons on a calorimeter with length of 10 λ_{int}
    3. Now do the same for 100 GeV pions.
    4. The cross section for interactions induced by 1 MeV neutrons in lead amounts to ~ 5b. Calculate the mean free path between subsequent interactions for neutrons and compare this to electrons (1 mm) and protons (10 µm) of the same energy. Note the impact on calorimeter design for detecting neutrons.
    5. On slide 50, calculate the energy per cascade nucleon after another generation of spallation. Use figure 31a to estimate the nb of protons and neutrons for 117 MeV.

1. Particle interactions with matter

1. The energy stored in one 7 TeV LHC beam is 362 MJ. How much energy will be released in the SHiP target per spill? Assume:
    - Beam intensity: $4*10^{13}$ protons per spill
    - Beam energy: 400 GeV

LHC: 2808 (nb of bunches)$*1.15*10^{11}$ (nb of protons/bunch)$*7*10^{12}$(7 TeV)$*1.602*10^{-19}$ J = 362 MJ

SHiP: $4*10^{13}*4*10^{11}$(400 GeV)$*1.602*10^{-19}$ J = 2.6 MJ

material damage can occur by beam induced thermal shock when energies reach the order of kJ/cm^3

1. Particle interactions with matter

2. What is the probability that a multi GeV photon interacts in 1 cm of lead? The same for carbon? How much energy will an electron lose? Calculate using the cross section, compare your answer with slide 19.

Lead: $\lambda = \frac{1}{N\sigma} = \frac{M}{\rho N_A \sigma}$ = (207.2 g/mol) / (11.35 g/cm^3 *6.022*10^{23}mol^{-1}*46*10^{-24} cm^2) = 0.65 cm vs 0.72

$$P(l) = \int_0^l p(l')\mathrm{d}l' = 1 - \exp\left(-\frac{l}{\lambda}\right)$$ = 1- exp (-1./0.65) = 0.79 vs 0.75

Carbon: (12 g/mol) / (2.26 g/cm^3 *6.022*10^{23}mol^{-1}*0.245*10^{-24} cm^2) = 44 cm. P(l) = 1-exp(-1./44.)=0.02

Energy loss electron in lead 1-exp(10./5.6)=0.83, in carbon 1-exp(10./188.)=0.05

1. Particle interactions with matter

3. How do 10 MeV photons interact in aluminium?

At 10 MeV in aluminium, Compton scattering is the dominating effect.

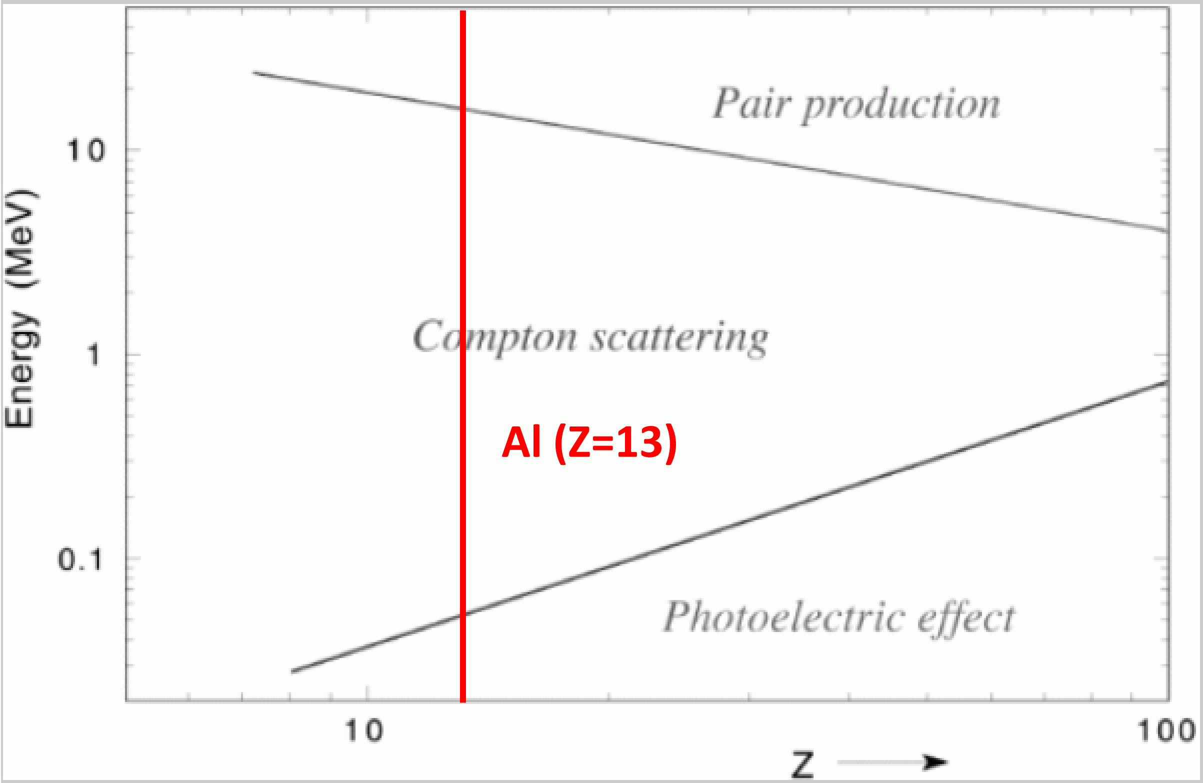


Fig. 2.7. The energy domains in which photoelectric effect, Compton scattering and pair production are the most likely processes to occur, as a function of the *Z* value of the absorber material.

1. Particle interactions with matter

4. Calculate the effective radiation length and Molière radius of the SHiP target, which consists of 54.1 cm of Mo, 91.5 cm of W, 5.4 cm of Ta, 8.5 cm of H_2O.

$$\frac{1}{X_0} = \sum_i V_i/X_i \rightarrow X_0 = 1/(1/159.5*(54.1/0.9593+91.5/0.3504+5.4/0.4094+8.5/36.08))=0.1909 \text{ cm}$$

$$\rho_M = \frac{\sqrt{4\pi\alpha}m_e c^2}{E_c}X_0 \approx \frac{21\text{ MeV}}{E_c}X_0 \rightarrow$$

$$\rho_M = 1/(1/159.5*(54.1/1.469+91.5/0.9327+5.4/1.073+8.5/9.768))= 1.13 \text{ cm}$$

2. Electromagnetic showers

1. Determine the number of X_0 required to contain 95% of 10 GeV electron showers in lead, copper and aluminium. Compare this with 99% containment.

$L(95\%) = t_{max}+0.08\ Z + 9.6\ X_0$:

Pb: $L(95\%) = 3.12 + 0.08*82 + 9.6 = 19.6\ X_0$, $L(99\%) = 23\ X_0$, $X_0(Pb) = 5.6$ mm

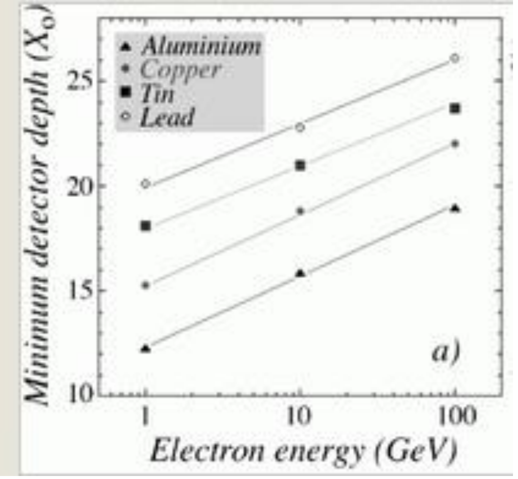
Cu: $L(95\%) = 2.71 + 0.08*29 + 9.6 = 14.6\ X_0$, $L(99\%) = 18\ X_0$, $X_0(Cu) = 14.3$ mm

Al: $L(95\%) = 2.37 + 0.08*13 + 9.6 = 13.0\ X_0$, $L(99\%) = 15\ X_0$, $X_0(Cu) = 89.0$ mm

2. How much extra X_0 are required to contain showers of 20 GeV photons?

A 20 GeV photon travels $9/7\ X_0 = 1.3\ X_0$ before converting into into an e^+e^- pair of 10 GeV each. It therefore takes $1.3\ X_0$ to contain twice as much em shower energy.

2. Electromagnetic showers



3. How much lead is required to absorb 90% of the energy contained by 100 GeV electrons?

25 X_0 contains 99% of the longitudinal shower = 14 cm

90% of the beam energy is contained in a cylinder with radius 1 ρ_M = 1.6 cm

thus a block of 5 * 5 * 15 * 11.35 = 4.3 kg of material would do it

4. What is the survival probability of 100 GeV muons after 300m of iron?

In iron, muons lose their energy at the rate of 1.1 GeV/m. So after 300m no muons are left.

5. Does an electromagnetic shower contain more electrons or positrons? Why?

More electrons since positrons are only produced in photon e^+e^- pair conversion.

2. Electromagnetic showers

6. Compare the Molière radius of copper (Z=29) to that of lead (Z=82) and discuss the impact on shower width in both materials.

Density (Cu) = 8.96 g cm^{-3} , density(Pb) = 11.35 g cm^{-3}.

X_0 (Cu) = 14.3 mm, X_0 (Pb) = 5.6 mm almost a factor of three difference due to difference in Z.

ρ_M (Cu) = 15.2 mm, ρ_M (Pb) = 16.0 mm because of no Z dependence.

As a consequence, the development of em showers in these two absorber materials has very different characteristics. In the longitudinal direction, it takes about three times as much copper as lead (in cm) to contain these showers. However, laterally, the showers in copper are even *narrower* than those in lead.

3. Hadronic showers

1. What is the major difference between em and hadronic showers?

Hadronic showers deposit a part their energy as “invisible energy” that can not be measured. This leads to a hadronic calorimeter response that is smaller than the electromagnetic one and a worse energy resolution for hadronic shower detection.

2. Calculate the *punch through* probability for 100 GeV protons on a 85 cm long Pb calorimeter with length of 10 λ_{int}

The *punch through* probability is the probability that a particle traverses a calorimeter without causing a nuclear reaction. It is thus $\exp(-850/\lambda_{int}) = \exp(-10) \sim 0.00005$

3. The same for 100 GeV pions.

λ_{int} (π) 25% greater than λ_{int} (p), hence $\exp(-850/1.25*\lambda_{int}) = \exp(-7) \sim 0.001$

3. Hadronic showers

4. The cross section for interactions induced by 1 MeV neutrons in lead amounts to ~ 5b. Calculate the mean free path between subsequent interactions for neutrons and compare this to electrons (1 mm) and protons (10 µm) of the same energy. Note the impact on calorimeter design for detecting neutrons.

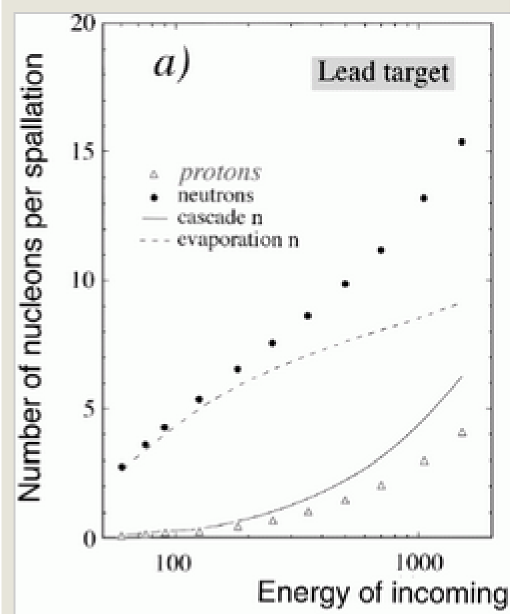
$$\lambda = \frac{1}{N\sigma} = \frac{M}{\rho N_A \sigma} = (207.2\ \mathrm{g/mol}) / (11.35\ \mathrm{g/cm^3} * 6.022 * 10^{23} \mathrm{mol^{-1}} * 5 * 10^{-24}\ \mathrm{cm^2}) \sim 0.6\ \mathrm{cm}$$

Distances travelled by evaporation neutrons are much larger than those of charged particles of the same energy. Because of this, they dominate the tails of shower profiles both lateral and longitudinal. Hence for calorimeters where neutron detection is important, the signal integration will have to take place over a larger volume.

3. Hadronic showers

5. On slide 50, calculate the energy per cascade nucleon after another generation of spallation. Use figure 31a to estimate the nb of protons and neutrons for 117 MeV.

- (Fig. 31a: 5.3 n, 0.25 p)*4.2=22.3 n, 1.1 p
- 1.1 p + (1.1*126/82=) 1.6 n cascade, 21 evaporation n
- 4.2 * 117 = 490 MeV; 23*7.9 = 185 MeV be; 62 MeV ke
- (490-185-62)=243 MeV shared by 1.1p+1.6n+recoil (3MeV): 89 MeV/nucleon



hadron (MeV)

Some remarks on the 6 problems

- You have one more week to send us solutions to problems 1 & 2
- Marked solutions will be returned
- Next deadline for submission of all solutions: 8 December
- Gold certificate for students who pass all problems and present a project (2 slides, 5 min presentation)
- Silver certificate for students who either pass all problems or present a project

Problem no. 4 Statistics of measurements

i. Invariant mass (see also lecture 1 (slide 27-28) and 2 (41-44))

- From the measured masses and momenta of decay products, reconstruct the mass of the initial particle
- Invariant mass = mass in rest frame, calculated from energy and momentum in any frame
- For a decay into 2 particles: $M^2 = (E_1 + E_2)^2 - \|\mathbf{p}_1 + \mathbf{p}_2\|^2$

ii. Error propagation (for a simple description see http://www.met.rdg.ac.uk/~swrhgnrj/combining_errors.pdf)

- If a = f(b), then the error in a is given by $\Delta\, a = \left|\frac{df(b)}{db}\right| \Delta\, b$
- If a = b + c, and $\Delta\, b$ and $\Delta\, c$ uncorrelated, then $\Delta\, a = \sqrt{(\Delta\, b)^2 + (\Delta\, c)^2}$

iii. Efficiency never 100%. Need to reduce the expected number by the (in)efficiency.

iv. Significance of signal

- For counting: Poisson statistics apply, can approximate by Gaussian with σ = sqrt(B)
- Signal/sqrt(B) (how likely is it that the background distribution extends to the signal)
- Power of signal = Signal/sqrt(B+S) (how strong is the signal in a critical region: discovery)

Problem 5. Conservation of lepton and baryon numbers

- Baryon and lepton numbers are internal symmetries
- Baryon number:
    - $B = \frac{1}{3}(n_q\text{-}n_{\bar{q}})$
    - Hadrons with a baryon number of **0** are called **mesons**
    - Particles **without quarks** have baryon number **0**
    - Hadrons with a baryon number of **+/- 1** are called **(anti)baryons**
    - Baryon number is conserved during nuclear interactions (proton decay?)
- Lepton number:
    - Spin ½ particles that do not undergo strong interactions:
    - $L = n_l\text{-}n_{\bar{l}}$
    - Total lepton number is conserved in SM (e.g. $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$)

Problem 5. Continued, some hints

- Remember decay width and lifetime are connected by the uncertainty principle
- Remember $G_F \sim 1/M_W^2$
- Compare the diagrams of muon decay to that of proton decay and divide amplitudes