# Problem 1. Relativistic kinematics

1. The meson predominantly decays into two photons.
2. Write down an expression for the energy of each photon in therest frame in terms of the mass *mπ*.

[2 marks]

1. One of the photons is produced at spherical polar angle with respect to the positive *z* axis in the rest frame. What is the corresponding spherical polar angle of the other photon?

[2 marks]

1. The Lorentz transformation for the energy of a particle when in a frame moving with velocity *v* along the *z* axis is given by

where .

1. In the laboratory frame, the has a velocity *v* along the *z* axis. Find an expression for the energies in this frame of each of the two photons described in part (i)(b). Hence, ﬁnd an expression for the sum of their energies and comment on the physical signiﬁcance of your result.

[5 marks]

1. Find the maximum and minimum possible values of the energy of either one of the photons in the laboratory frame. What angles do these values correspond to in the rest frame?

[4 marks]

1. The has no spin so there is no preferred direction for the photons in its rest frame. Hence the photons are emitted isotropically and so have a uniform distribution in the variable *ε*, where . Hence, or otherwise, show that the probability distribution for the energy of either one of the photons in the laboratory frame is

for energies *E’* between the maximum and minimum values.

[3 marks]

1. Sketch the probability distribution given in part (ii)(c), marking the maximum and minimum possible photon energies and the mean photon energy. Also show the value of your answer to part (i)(a) on the sketch.

[4 marks]

# Problem 2 Cross section and interaction probability

1. The cross-section for a two-particle reaction is the effective area for the reaction to occur presented to an incoming particle by the target particle. Consider a box with a front face area and length containing target nuclei with a number density of nuclei per unit volume. Each nucleus has a cross-section . By considering particles normally incident on the front face, and assuming the target density is low enough that the targets do not obscure each other, show that the probability that the incoming particle will react is given by .

[4 marks]

1. For high enough antineutrino energies , the interaction cross-section with protons is approximately given by

where is the weak interaction Fermi constant. For antineutrinos with an energy of 10 MeV, use the information given at the end of the question to evaluate this cross section in units of m2.

[3 marks]

1. Antineutrinos can interact through the above reaction with the protons in the hydrogen atoms of water molecules. Given that 1 litre of water has 1 kg mass, show that an approximate value for the target density for water is .

[4 marks]

1. The Kamiokande-II detector contained 2 ktonnes of water. Assuming the water container to be cubic, show that the probability for an antineutrino of energy 10 MeV entering perpendicularly through one side of the cube to react in the water is approximately .

[3 marks]

1. Kamiokande-II detected 11 antineutrino reactions in coincidence with the explosion SN1987A, which was a supernova in the Large Magellanic Cloud, 150,000 light years away. Assuming all the antineutrinos had energies of around 10 MeV and the supernova explosion was isotropic, estimate the total number of neutrinos emitted by the supernova. Assuming most of the energy was released through antineutrinos, estimate the total energy emitted by the supernova explosion.

[4 marks]

1. The gravitation energy of a uniform sphere of radius *r* and mass *m* is given by

Compare the value of the supernova energy with the gravitational energy released by the collapse of a star with a mass approximately the same as the Sun. Assume the star collapses down to a sphere of nuclear density from an initial radius very much larger than the ﬁnal radius.

[2 marks]

[Total 20 marks]

*The Fermi constant is:* .

*The most common isotope of oxygen is: .*

*The mass of the Sun is: .*

*The volume of a nucleon can be taken to be: .*

# Problem 3. Momentum resolution

1. Write down the expression for the Lorentz force on a charged particle moving with velocity in a uniform magnetic ﬁeld, carefully deﬁning all the quantities. Assuming the magnetic ﬁeld is in the -direction in the Cartesian coordinate system, derive the equations of motion in terms of each velocity component and show that the form is similar to simple harmonic motion (you may assume the particle’s motion is non-relativistic).

[5 marks]

1. Show that the solutions to the equations of motion can be given by

where , and are constants and is time. Find an expression for and sketch the trajectory of the particle, labelling all relevant quantities. Describe how the trajectory of the particle would change if it passed through a dense material and lost kinetic energy.

[7 marks]

1. In particle physics experiments, the bending of charged particles in the -plane is used to measure their charge and the component of their momentum transverse to the magnetic ﬁeld is given by

where is the radius of curvature measured in metres, in GeV/c and in Tesla. In practice the sagitta s is measured as shown in the ﬁgure. Show that the transverse momentum can be approximated by

[3 marks]

Diagram

Description automatically generated

1. Using the expression from part (iii) and assuming an uncertainity on the sagitta measurement, derive an expression for the relative uncertainty on the measured transverse momentum and comment on its dependence on , and .

[2 marks]

1. The Babar experiment at the Stanford Linear Collider Center has a tracking detector with a radius of in a 1.5 T magnetic ﬁeld and can measure the sagitta to an accuracy of . Calculate the expected fractional uncertainty on a 5 GeV/c transverse momentum measurement.

# Problem 4. Statistics of measurements

This question addresses statistics of measurements as applied to the detection of the Higgs boson through its decay to two photons. In the following, you may take the mass of the Higgs boson to be 125 GeV. The CMS and ATLAS experiments at the Large Hadron Collider use large electromagnetic calorimeters to detect decay products of the Higgs boson and to measure its mass.

1. In particle physics, the invariant mass of the system is equal to the mass in the rest frame and can be calculated from the system energy and its momentum as measured in any frame, by the energy-momentum relation

in natural units where . Show that the invariant mass, , of two photons is given by where and are the energies of the two photons and is the angle between the directions of ﬂight of these photons. You may rewrite the energies of the two photons in terms of their momenta.

[5 marks]

1. Assuming the angular resolution is accurate enough that its error can be neglected, show that the fractional mass resolution is given by

where is the error on the two photon invariant mass, and are the errors on the photon energies .

[5 marks]

1. The fractional calorimeter energy resolution is given by

where energy is measured in GeV and the coefﬁcients a and b depend on the calorimeter construction: is ∼ 3% for the CMS calorimeter and ∼ 10% for the ATLAS calorimeter. The coefﬁcient is ∼ 0.5% for both calorimeters. Consider the Higgs boson decaying into two photons of equal energies = = 50GeV. Estimate the absolute mass resolution in GeV for the Higgs mass reconstructed in the CMS and ATLAS calorimeters.

[4 marks]

1. At the LHC, the Higgs signal appears as a Gaussian peak in the distribution, with the mean = 125 GeV and = 1.3 GeV, on top of the uniformly distributed background from other processes. The background contributes random combinations of photons at the rate of 60000 events per each GeV in the range of the Higgs mass. Assume that the raw number of produced Higgs is 5000 events and the total efﬁciency for reconstructing a photon in the calorimeter is 60%. Estimate the signiﬁcance of the Higgs signal if events within ± 1 of the Higgs mass are accepted.

[6 marks]

# Problem 5. Conservation of lepton and baryon numbers

This question addresses the conservation of lepton and baryon numbers in particle physics, proton stability and its relation to the lifetime of human beings.

1. Give the lepton and the baryon numbers for fundamental fermions (leptons and quarks) in the Standard Model (SM). Give two examples of decays forbidden in the SM, one violating the lepton number and one violating the baryon number. Draw the Feynman diagram for the leptonic muon decay, labelling all lines.

[3 marks]

1. The leptonic width of muon decay is given by , where , and the muon mass µ = 0.106 GeV. Calculate the muon lifetime. You may assume thatis the only signiﬁcant decay mode of the muon.

[3 marks]

1. Explain why must be proportional to by use of dimensional grounds. Note that the amplitude of the decay is *∼ 1/MW2*, where *MW*  is the mass ofthe W boson.

[3 marks]

1. Explain why protons are stable particles in the Standard Model. Most Grand Uniﬁed Theories explicitly break baryon number symmetry but require the conservation of the difference *B-L* of baryon, *B*, and lepton, *L*, numbers. Explain why the decay is allowed in such theories. Draw the simplest Feynman diagram for this decay assuming that the decay is mediated by a new boson X, which can couple to both leptons and quarks.

[3 marks]

1. If the mass of the X boson is, estimate the proton lifetime given that the muon lifetime is *τµ* = 2.2 *µ*s and the mass of W boson is 80 GeV. You may assume that the proton decays exclusively to the ﬁnal state and that the X boson has conventional weak couplings to the SM fermions.

[4 marks]

1. The best experimental limits on the proton lifetime are in the range *τp* > 1031-1033 years depending on the model. The proton lifetime can also be estimated from the requirement that the total radiation dose inside a human body does not exceed the annual dose delivered by cosmic ray muons, 30 mrad (1rad = 6.2 x 1010 MeV/kg). Calculate the value of the mean proton lifetime that would result in this dose, if protons underwent decay and their total mass energy (0.94 GeV) then appeared in the form of ionising radiation inside the human body. Comment on the signiﬁcance of your result. You may assume the same number of protons and neutrons per 1 kg of the human body and that kg.

[4 marks]

# Problem 6. Lepton flavour violation

The highest priority of particle physics experiments is to search for phenomena that cannot be explained by the Standard Model. Here searches for Lepton Number and Lepton Flavour Violation (LNV and LFV) are the two important topics of investigation. Many models predict enhanced LFV effects that manifest themselves in the 3rd generation of leptons, e.g. tau decays.

1. State the leptons in the Standard Model. Give a short summary of their quantum numbers and an approximate mass hierarchy of the charged leptons and neutrinos.
   1. Give one example of semileptonic decay and one example of purely leptonic decay of the tau lepton.
   2. Give one example of semileptonic LNV and one example of purely leptonic LFV decay of the tau lepton.
   3. Can muons decay into semileptonic ﬁnal states containing hadrons?

[8 marks]

1. Using possibly qualitative considerations explain
   1. why the branching fraction of the decay BR( ) is a factor of 5 larger than BR(;
   2. why the ratio of purely leptonic to semileptonic decays of the tau lepton is ~2/3.

Justify your answer using appropriate Feynman diagrams. (Note that the tau lepton is lighter than the lightest charmed meson).

[6 marks]

1. The decay width of the decay is given by . Assuming *µ-τ* universality of the weak charged-current and that BR() = 100%, and BR() = 20% calculate the lifetimes of the muon and tau leptons. Tau leptons are copiously produced in the electron-positron collisions at the KEKB accelerator with a typical energy of 3 GeV, and in the proton collisions at LHC with a typical energy of 100 GeV. The world average value of the measured lifetime of the tau lepton is 2.97 x 10 -13 s. Calculate the typical travel distances of tau leptons before decay at KEKB and at LHC. Comment on the vertex detectors required at KEKB and at LHC to reconstruct the decay vertices of tau lepton.

[6 marks]

[Total 20 marks]