Energy measurement in particle physics

Video of the lecturer

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Calorimetry in Particle Physics

Video of the lecturer

This lecture draws a lot from the following references:

the Review Article ‘Calorimetry for Particle Physics’,
C.W. Fabjan and F. Gianotti, Rev. Mod. Phys., Vol. 75, N0. 4, October 2003

the Monograph ‘Calorimetry, Energy Measurement in Particle Physics’,
R. Wigmans, Oxford University Press, Second edition, 2017

the Training lectures at CERN by Werner Riegler

Bremsstrahlung

Video of the lecturer

A charged particle of mass M and charge $q=Z_1 e$ is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and it therefore radiates → Bremsstrahlung.

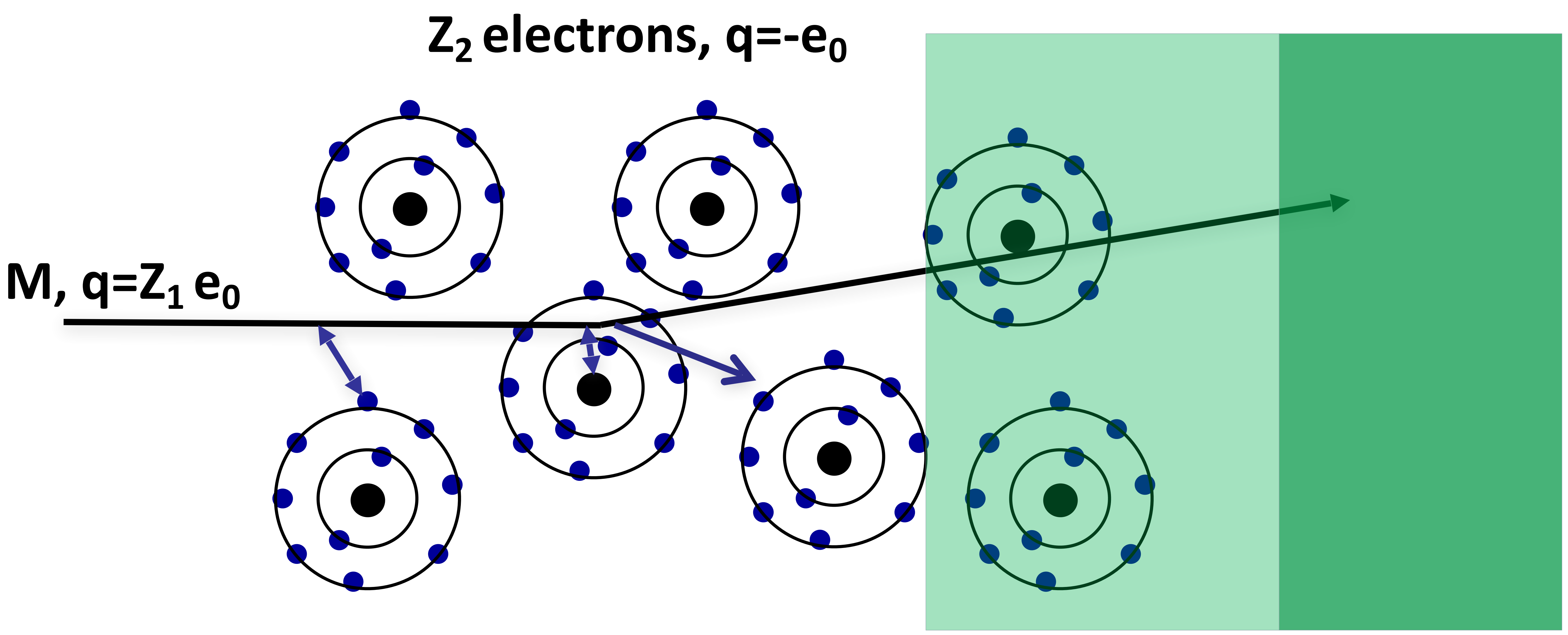
From Bethe's theory the elastic scattering off the Nucleus is given by

$$\epsilon_0(q) = Z_2 - \sum_{j=1}^{Z_2} \int e^{i(\vec{q}\vec{r}_j)} \psi_0^2(\vec{r}_j) d^3 r_1 \ldots d^3 r z_2 = Z_2 - F(q)$$

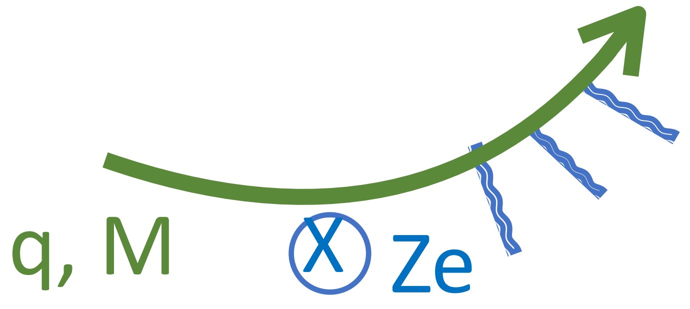
$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \frac{Z_1 (Z_2 - F) e_0^2}{2pv} \right)^2 \frac{1}{\sin^4 \theta/2}$$

where F(q) describes the partial shielding of the nucleus by the electrons.

Effective values for F are used in the following expressions.



Bremsstrahlung: Classical approach



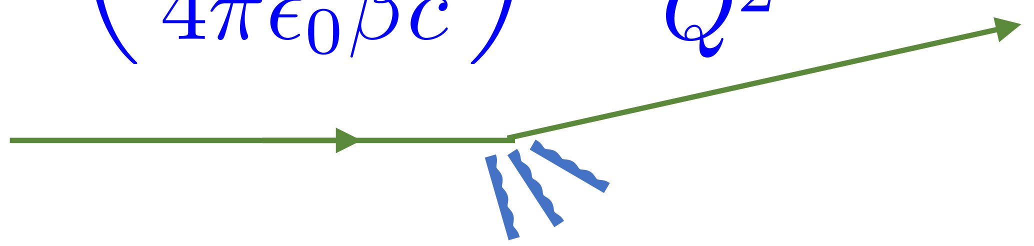
$$q = Z_1 \cdot e$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Z_1Z_2e^2}{4\pi\epsilon_0\; p \cdot v}\right)^2 \frac{1}{\left(2\sin\frac{\theta}{2}\right)^4} \qquad \boxed{p = Mv\gamma}$$

"Rutherford Scattering"

in Terms of Momentum Transfer: $Q^2 = 2p^2(1-\cos\theta)$

$$\frac{d\sigma}{dQ} = 8\pi\left(\frac{Z_1Z_2e^2}{4\pi\epsilon_0\beta c}\right)^2 \cdot \frac{1}{Q^2}$$



$$Q = |\vec{p} - \vec{p}'|$$

$$\lim_{\omega\to 0}\frac{dI}{d\omega} \sim \frac{2}{3\pi}\frac{Z_1^2e^2}{M^2c^3}\frac{Q^2}{4\pi\epsilon_0}$$

Radiated energy between ω and ω'

$$\frac{dE}{dx} = \frac{N_A\rho}{A}\cdot\int_0^{\omega_{max}} d\omega \int_{Q_{min}}^{Q_{max}} dQ \frac{dI}{d\omega}\cdot\frac{d\sigma}{dQ} \qquad \omega_{max} = \frac{E}{\hbar}$$

$$\frac{dE}{dx} = \frac{N_A\rho}{A}\cdot\frac{16}{3}\alpha\cdot Z^2\cdot\left(\frac{Z_1^2e^2}{4\pi\epsilon_0 Mc^2}\right)^2\cdot E\cdot\ln\frac{Q_{max}}{Q_{min}}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \sim \frac{1}{137}$$

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A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of Charge Ze.

Because of the acceleration the particle radiates EM waves → energy loss.

Coulomb-Scattering (Rutherford Scattering) describes the deflection of the particle.

Maxwell's Equations describe the radiated energy for a given momentum transfer.

→ dE/dx

Bremsstrahlung: Quantum Mechanics

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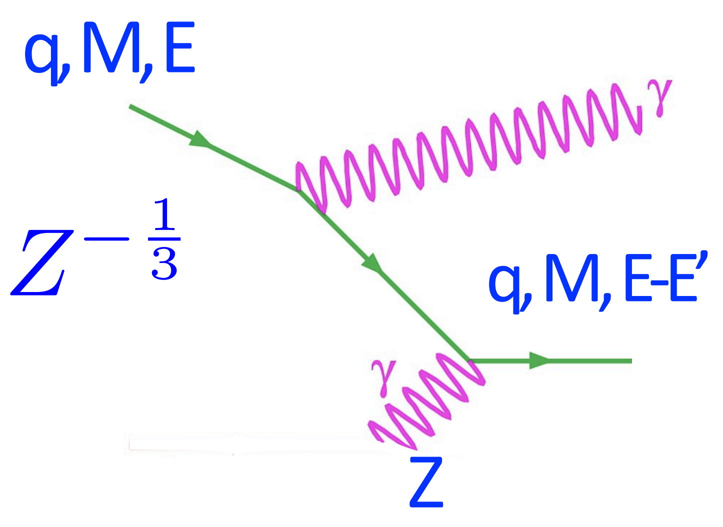
$$q = Z_1 e_1,\ \ E + Mc^2 >> 137 Mc^2 Z^{-\frac{1}{3}}$$



→ Highly Relativistic:

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 \frac{1}{E'} F(E,E')$$

$$F(E,E') = \left[1 + \left(1 - \frac{E'}{E+Mc^2}\right)^2 - \frac{2}{3}\left(1 - \frac{E'}{E+Mc^2}\right)\right] \ln 183 Z^{-\frac{1}{3}} + \frac{1}{9}\left(1 - \frac{E'}{E+Mc^2}\right)$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A}\int_0^E E' \frac{d\sigma}{dE'} dE' \sim 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 E \left[\ln 183 Z^{-\frac{1}{3}} + \frac{1}{18}\right]$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A} 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 E \ln\left(183 Z^{-\frac{1}{3}}\right)$$

$$E(x) = E_0 \exp^{-\frac{x}{X_0}}$$

$$X_0 = \frac{A}{4\alpha N_A \rho Z^2 \left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{Mc^2}\right)^2 \ln\left(183 Z^{-\frac{1}{3}}\right)}$$

X_0 radiation length

Proportional to Z^2/A of the Material.

Proportional to Z_1^4 of the incoming particle.

Proportional to ρ of the material.

Proportional $1/M^2$ of the incoming particle.

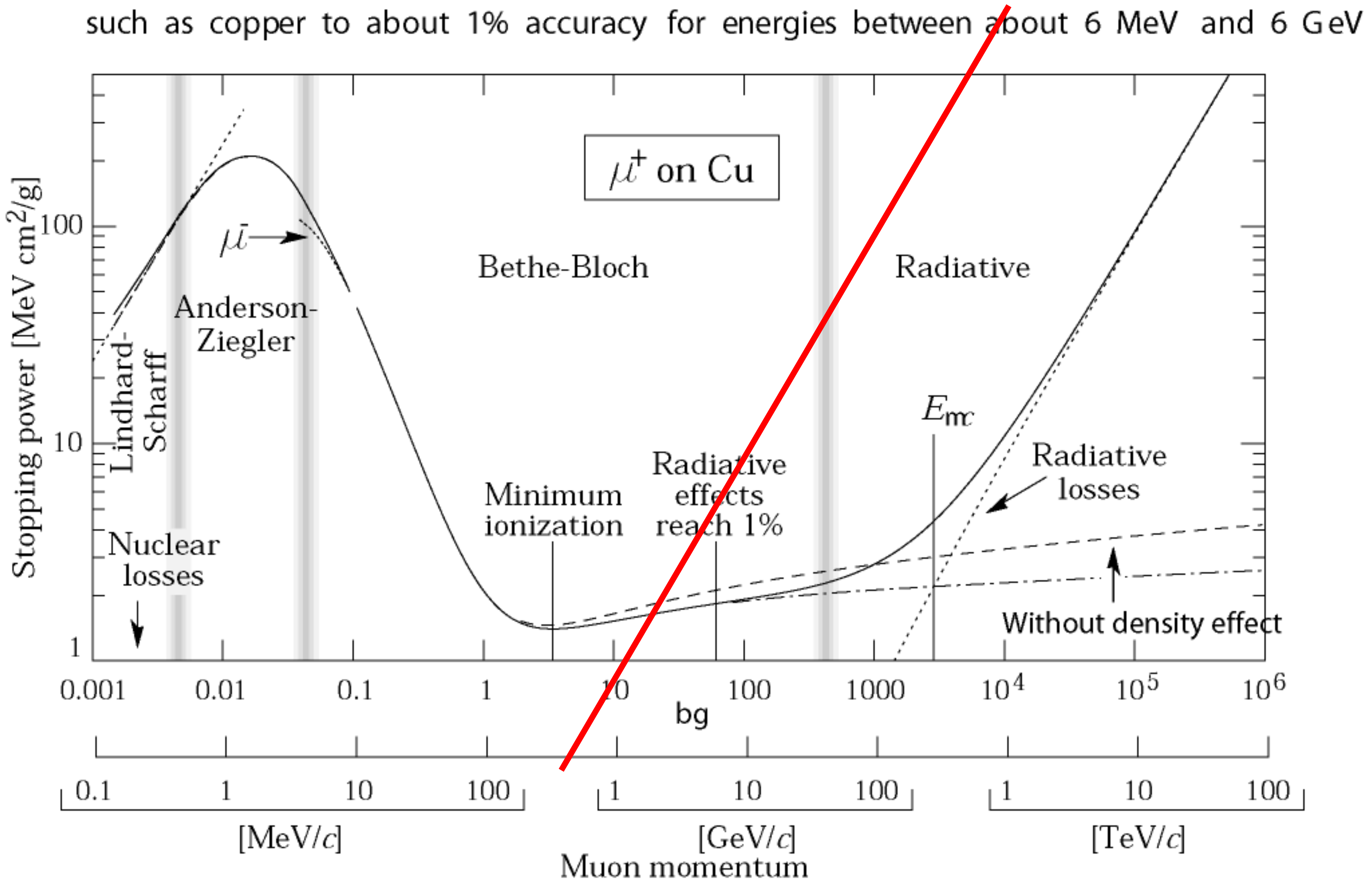
Proportional to the Energy of the Incoming particle →

$E(x)=e(-x/X_0)$ – 'Radiation Length'

$X_0 \propto M^2 A/ (\rho\ Z_1^4\ Z^2)$

X_0: Distance where the Energy E_0 of the incoming particle decreases $E_0\, e^{-1}=0.37E_0$

Critical Energy



Electron Momentum 5 50 500 MeV/c

Critical Energy: If dE/dx (Ionization) = dE/dx (Bremsstrahlung)

Electron in Copper: p ≈ 20MeV

Muon in Copper: p ≈ 400GeV

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For the muon, the second lightest particle after the electron, the critical energy is at 400GeV.

The EM Bremsstrahlung is much less important for muons. At the LHC and in cosmic-rays experiments can be relevant

Pair Production: Quantum Mechanics

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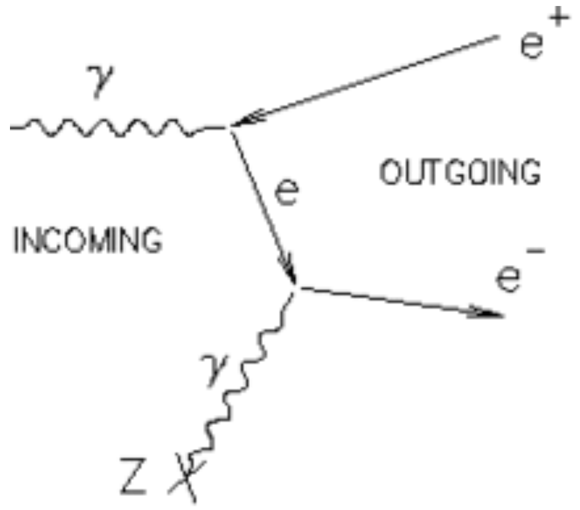
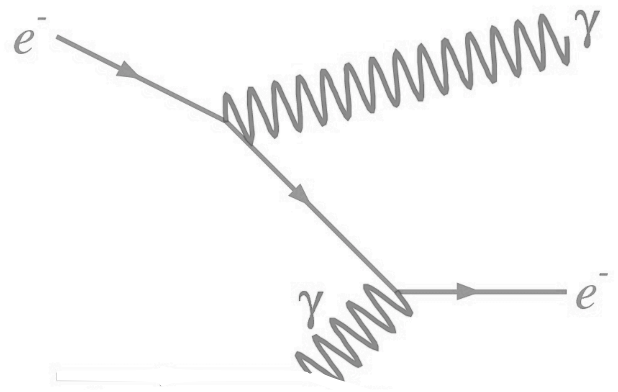


$$\gamma + \text{Nucl.} \rightarrow e^+ + e^- + \text{Nucl.}$$

The diagram is very similar to Bremsstrahlung



$$e^- + \text{Nucl.} \rightarrow \gamma + e^- + \text{Nucl.}$$

Crossing Symmetry: same cross-section

For $E\gamma >> m_e c^2 = 0.5\text{MeV}$: $\lambda = 9/7 X_0$

Average distance a high energy photon has to travel before it converts into an e^+e^- pair is equal to 9/7 of the distance that a high energy electron has to travel before reducing its energy from E_0 to $E_0 * e^{-1}$ by photon radiation.

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 r_e^2 \frac{1}{E} \cdot G(E,E') \qquad E >> 137 m_e c^2 Z^{-\frac{1}{3}}$$

$$G(E,E') = \left[\left(\frac{E' + m_e c^2}{E}\right)^2 r \left(1 - \frac{E' + m_e c^2}{E}\right)^2 + \frac{2}{3}\frac{E' + m_e c^2}{E}\left(1 \frac{E' + m_e c^2}{E}\right) \ln Z^{-\frac{1}{3}} \right]$$

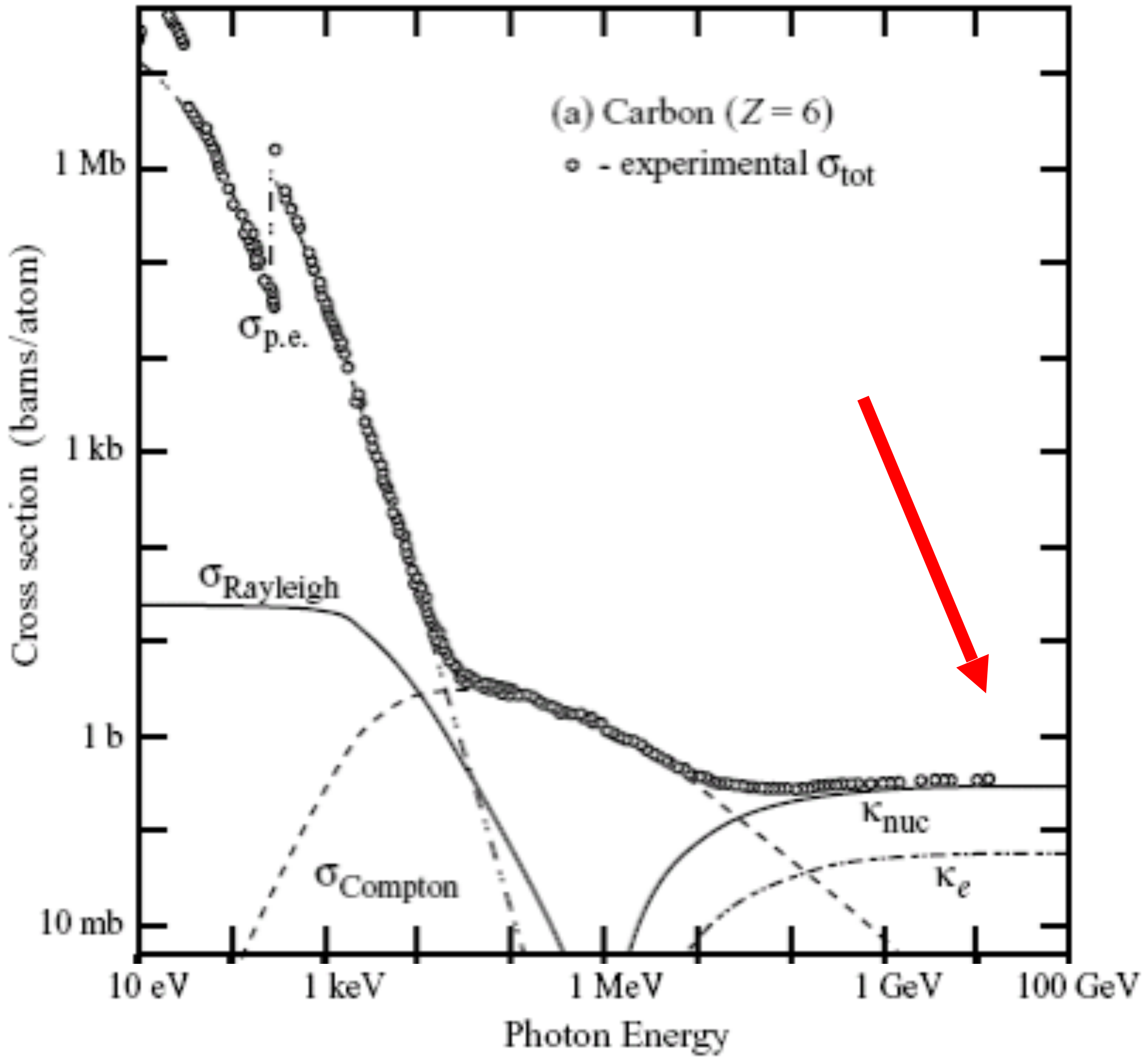
$$\left[-\frac{1}{9}\left(\frac{E' + m_e c^2}{E}\right)\left(1 - \frac{E' + m_e c^2}{E}\right)\right]$$

$$\sigma = \int_0^{E - 2m_e c^2} \frac{d\sigma'}{dE'} dE' = 4\alpha Z^2 r_e^2 \cdot \frac{7}{9} \ln 183 Z^{-\frac{1}{3}}$$

$$P(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}$$

$$\lambda = \frac{A}{\rho N_A \sigma} = \frac{9}{7} X_0$$

Probability that Photon converts to e^+e^- after a distance x



Bremsstrahlung + Pair Production → EM Shower

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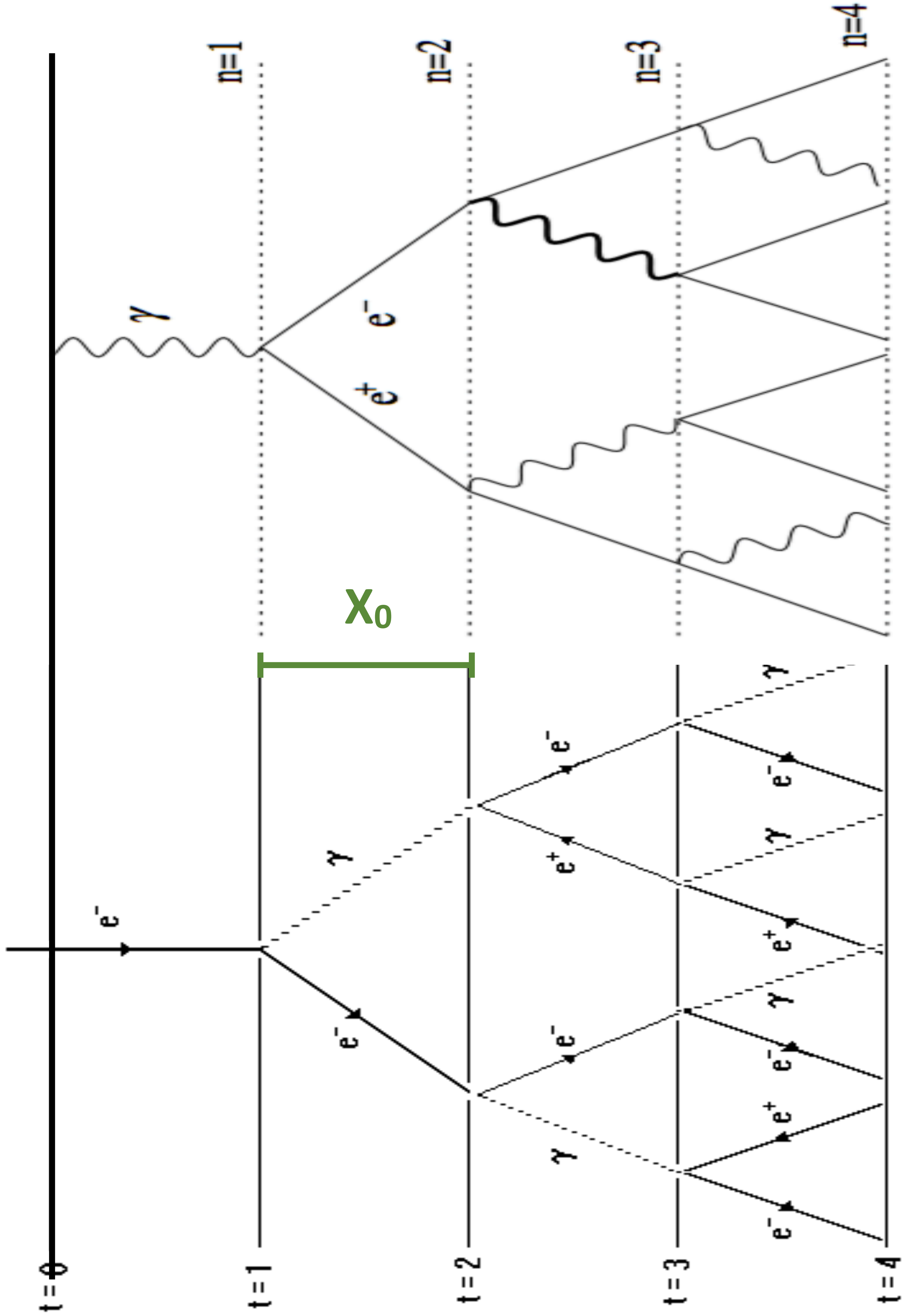

Electromagnetic Shower → EM Calorimeter

Electromagnetic shower of high-energy electrons and photons

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$$N(n) = 2^n$$

Number of particles ($e^{\pm}$,γ) after nX_0

$$E(n) = \frac{E_0}{2^n}$$

Average Energy of particles after nX_0

Shower stops if $E(n) = E_c$

$$n_{max} = \frac{1}{\ln 2} \ln \frac{E_0}{E_c}$$

Shower length increases with $\ln E_0$

$$N_{tr}(n) = 2^n$$

Number of $e^{\pm}$ track segments of length X_0 after nX_0

$$L = \sum_{n=0}^{n_{max}-1} 2^n X_0 = \left(2\frac{E_0}{E_c} - 1\right) X_0 \sim 2\frac{E_0}{E_c} X_0 = c_1 \cdot E_0$$

Total (charged) track length is proportional to the particle energy → Calorimeter Principle

Calorimetry: Energy Measurement by total Absorption of Particles

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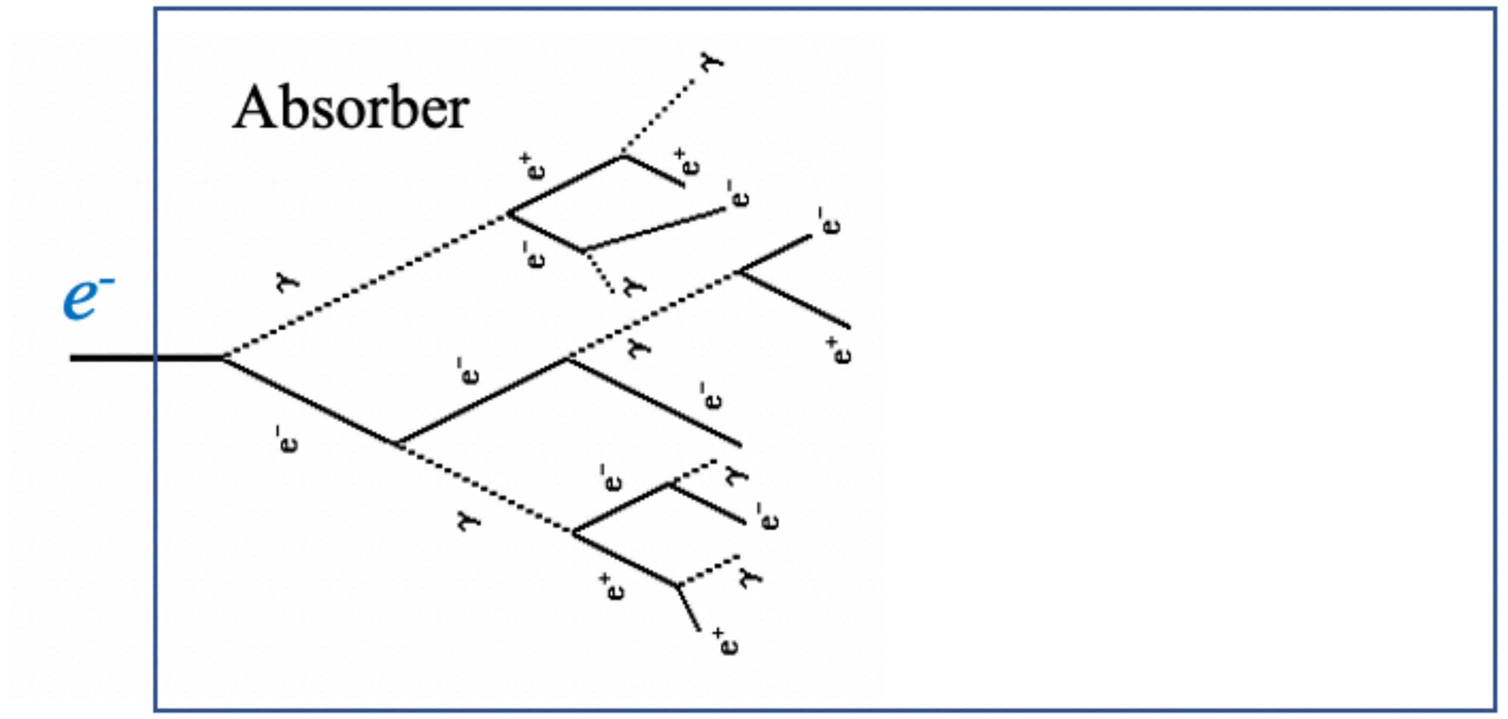
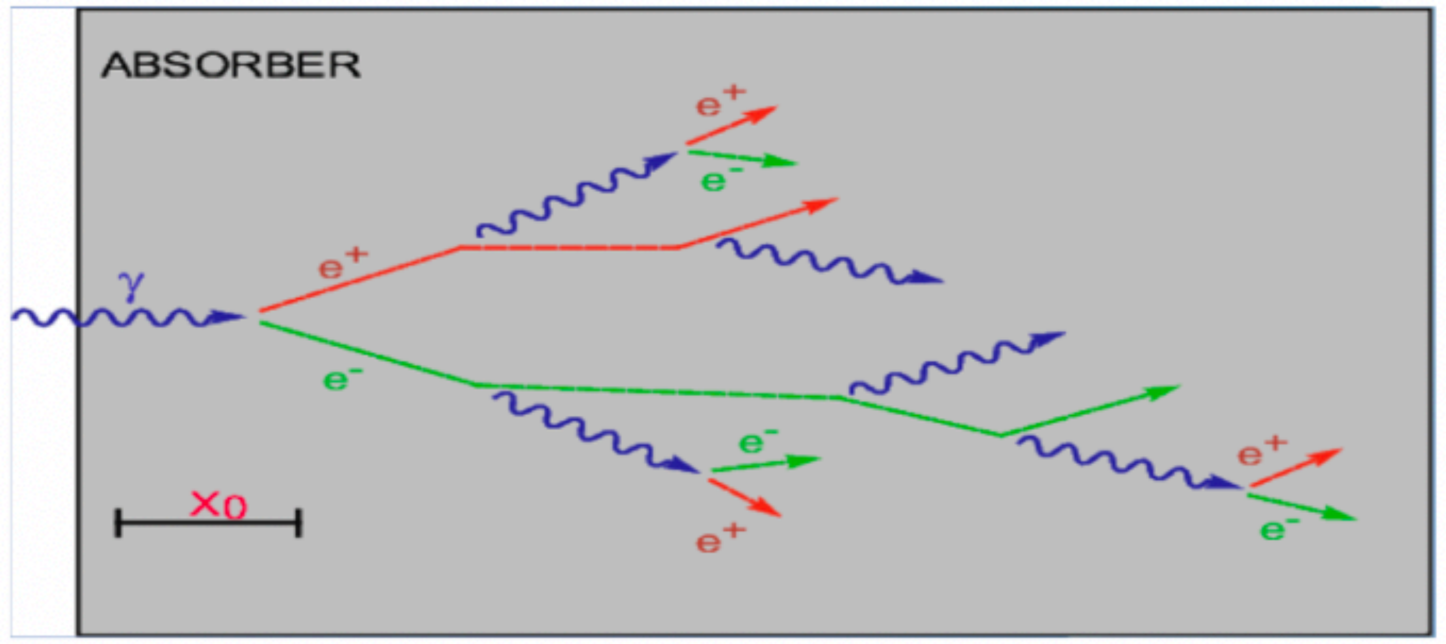




The $e^{\pm}$ in the Calorimeter ionize and excite the material

Ionization: e^-, I^+ pairs in the material

Excitation: Photons in the material

Measuring the total number of e^-, I^+ pairs or the total number of photons gives the particle Energy

If N is the total number of e^-, I^+ pairs or photons, $N = c_1 E_0$:

$$\Delta N = \sqrt{N} \quad \text{(Poisson statistics)}$$

$$\frac{\Delta E}{E} = \frac{\Delta N}{N} = \frac{1}{\sqrt{N}} = \frac{a}{\sqrt{E}}$$ → Radiation

Only Electrons and High Energy Photons show EM cascades at current GeV-TeV level Energies.

Strongly interacting particles like Pions, Kaons, produce hadronic showers in a similar fashion to the EM cascade → Hadronic calorimetry

Calorimetry: Energy Measurement by total Absorption of Particles

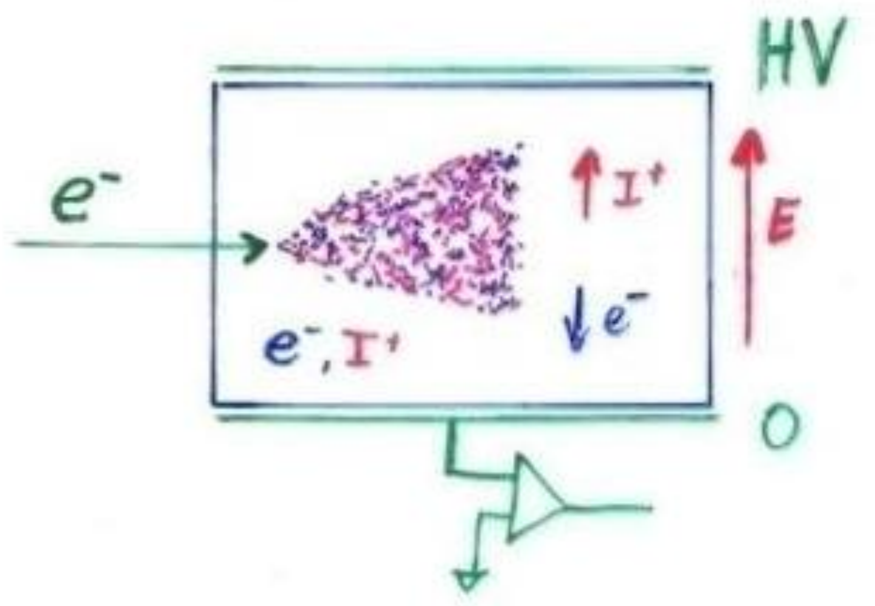
Video of the lecturer

The measurement is destructive. The particle cannot be studied further

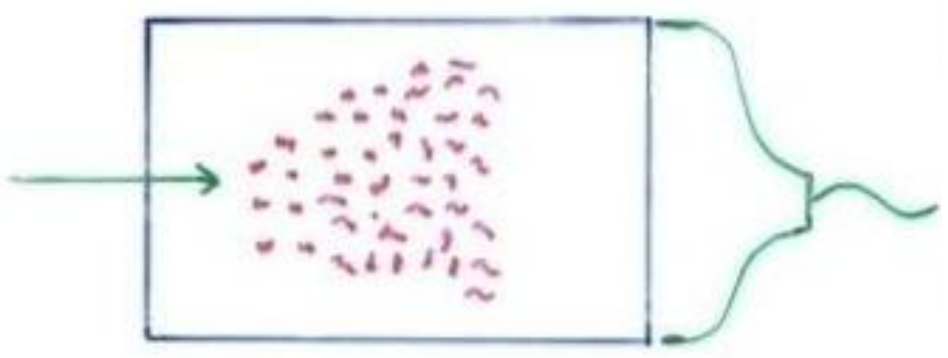
Energy measurement by:

Liquid Noble Gas

(Noble Liquids)



Collecting the produced charge



Measuring the photons produced by the collisions of the $e^{\pm}$ with the atom electrons of the material

Scintillating Crystals,

Plastic Scintillators

Total amount of pairs or photons is proportional to the total track length and is proportional to the particle energy

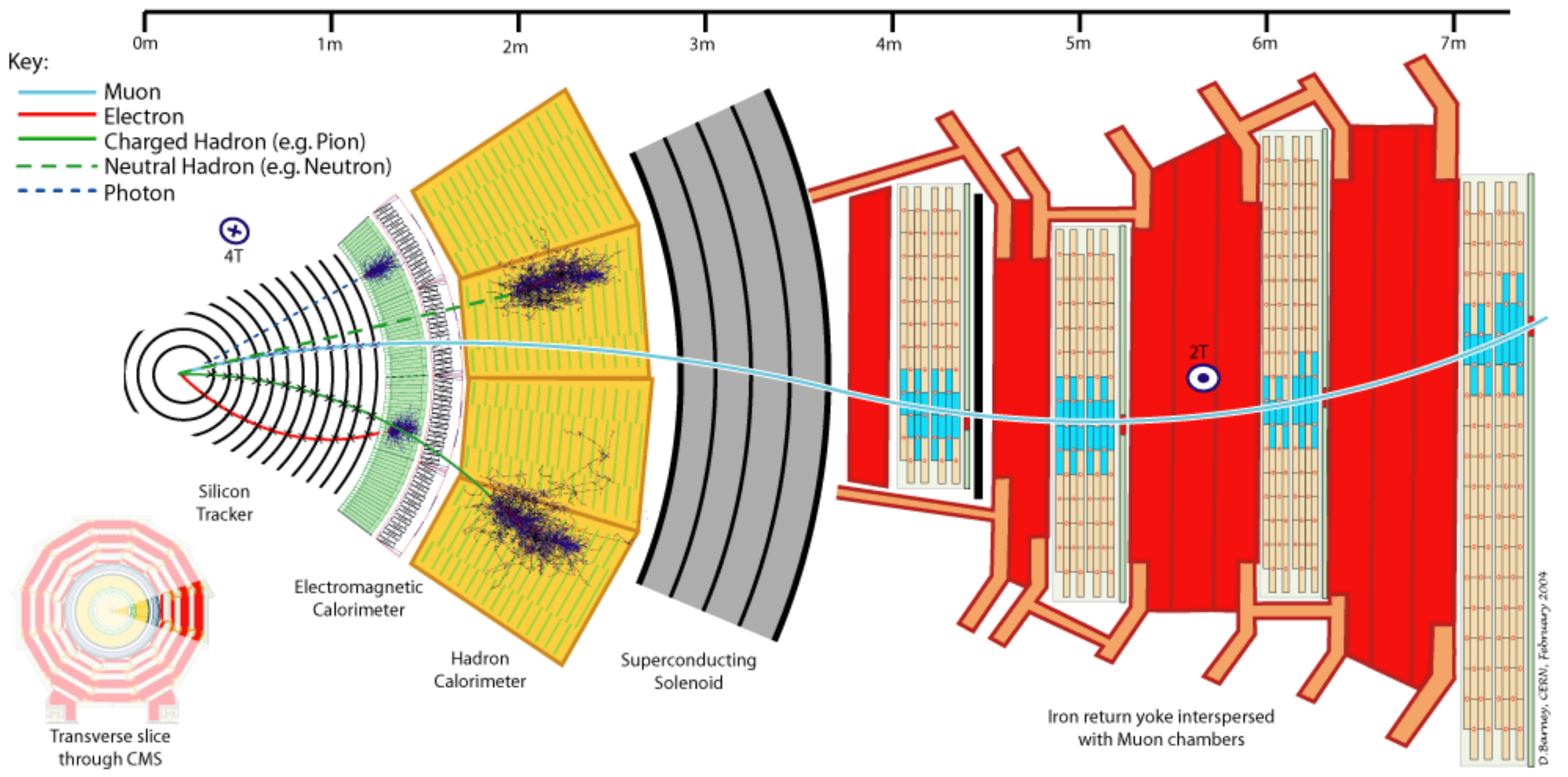
Calorimetry

Video of the lecturer

Calorimeters are blocks of instrumented material where particles are fully absorbed and their energy transformed into a measurable quantity.

The interaction of the incident particle with the detector (through electro-magnetic or strong processes) produces a shower of secondary particles with progressively degraded energy.

The energy deposited by the charged particles of the shower in the active part of the calorimeter, detected in the form of either charge or light, serves as a measurement of the energy of the incoming particle.



C.W. Fabjan and F. Gianotti, Rev. Mod. Phys., Vol. 75, N0. 4, October 2003

Calorimetry

Video of the lecturer

Calorimeters can be classified into:

- Electromagnetic Calorimeters,
  measure electrons and photons through their EM interactions.
- Hadron Calorimeters,
  measure hadrons through their strong and EM interactions.

The construction can be classified into:

- Homogeneous Calorimeters,
  built of only one type of material that performs both tasks, energy degradation and signal generation.
- Sampling Calorimeters,
  consist of alternating layers of an absorber, a dense material used to degrade the energy of the incident particle, and an active medium that provides the detectable signal.

C.W. Fabjan and F. Gianotti, Rev. Mod. Phys., Vol. 75, N0. 4, October 2003

Calorimetry

Video of the lecturer

Calorimeters are attractive in our field for different reasons:

Unlike magnet spectrometers where the momentum resolution worsens linearly with the particle momentum, typically the $\frac{\Delta E}{E} \propto \frac{1}{\sqrt{E}}$, where E is the particle energy —> very well suited for high-energy physics experiments.

Moreover, calorimeters are sensitive to all types of particles, charged and neutral. They can provide indirect detection of neutrinos and their energy through a measurement of the event missing energy.

Calorimeters are commonly used for trigger purposes since they can provide fast signals, easy to process and interpret.

They are space and therefore cost effective. Since the shower length $\propto$ ln (E) —> the detector thickness $\propto$ ln (E). On the contrary, for a fixed momentum resolution, the bending power BL^2 of a magnetic spectrometer must increase linearly with the particle momentum.

C.W. Fabjan and F. Gianotti, Rev. Mod. Phys., Vol. 75, N0. 4, October 2003

Interaction of Particles with Matter

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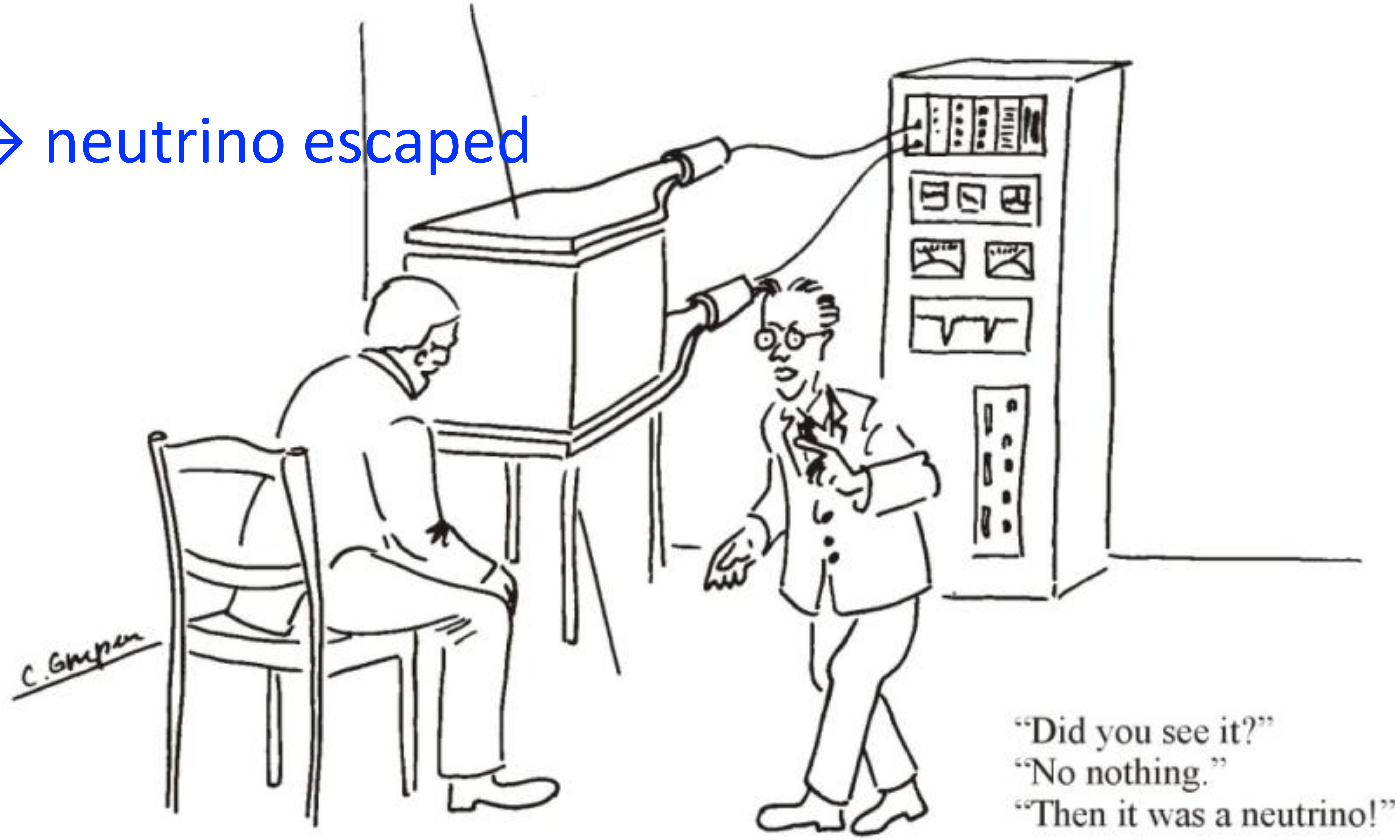
Detecting a particle requires its interaction with the detector

Neutrinos are directly seen in this way (see previous lecture)!

Neutrinos can also be measured by missing transverse energy.

E.g. p p collider $E_T=0$,

If the $\sum E_T$ of all collision products is ≠0 → neutrino escaped



Claus Grupen, Particle Detectors, Cambridge University Press, 1996

EM Calorimetry

Video of the lecturer

Approximate longitudinal shower development

$$N(n) = 2^n$$

Number of particles ($e^{\pm}$,γ) after nX_0

$$E(n) = \frac{E_0}{2^n}$$

Average Energy of particles after nX_0

Shower stops if $E(n) = E_c$

$$n_{max} = \frac{1}{\ln 2} \ln \frac{E_0}{E_c}$$

Shower length rises with $\ln E_0$

X_0 and ρ_m are two key parameters for the choice of calorimeter materials

Approximate transverse shower development

The transverse shower direction is mainly related to the Multiple Coulomb Scattering of the low Energy Electrons

$$\theta_0 \sim \frac{21(mrad)}{\beta p\,(MeV)} Z_1 \sqrt{\frac{X}{X_0}}$$

Electrons: $E_c, \; E \sim p \cdot c$

$$Z_1 = 1, \; \beta \sim 1$$

$$\theta_0 \sim \frac{21(mrad)}{\beta E_c\,(MeV)} Z_1 \sqrt{\frac{X}{X_0}}$$

$$E_c \sim \frac{610}{Z + 1.24} MeV \sim \frac{610}{Z} MeV$$

$$\theta = 0.0344 \cdot Z \cdot \sqrt{\frac{X}{X_0}}$$

Moliere Radius ρ_m = Lateral Shower Radius after $1X_0$

$$\rho_m \approx 0.0344 \cdot Z \cdot X_0$$

95% of Energy is in a Cylinder of $2\rho_m$ radius

Simulated EM Shower Profiles in $PbWO_4$

Video of the lecturer

Simulation of longitudinal shower profile **Simulation of transverse shower profile**

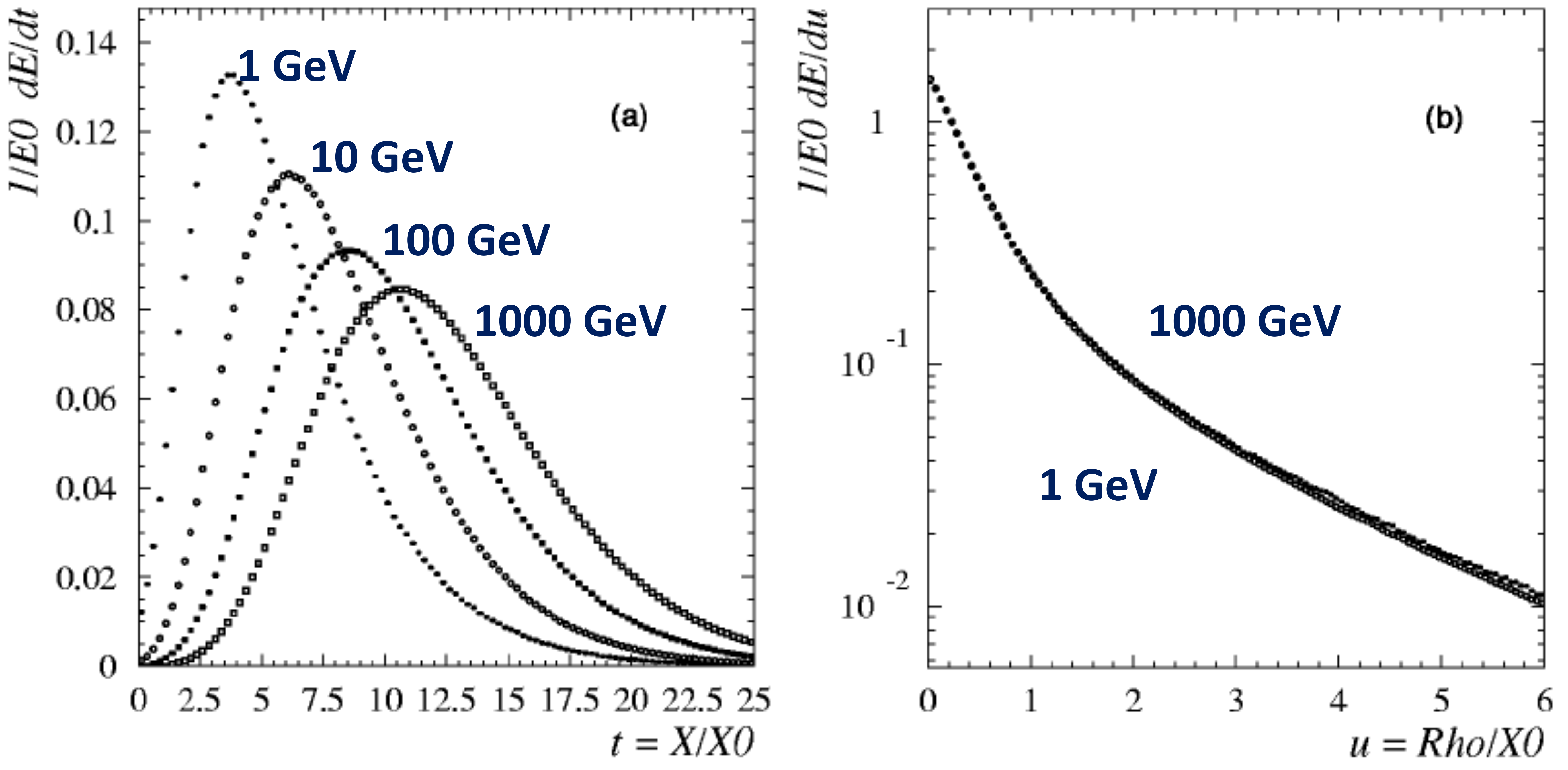


FIG. 2. (a) Simulated shower longitudinal profiles in $PbWO_4$, as a function of the material thickness (expressed in radiation lengths), for incident electrons of energy (from left to right) 1 GeV, 10 GeV, 100 GeV, 1 TeV. (b) Simulated radial shower profiles in $PbWO_4$, as a function of the radial distance from the shower axis (expressed in radiation lengths), for 1 GeV (closed circles) and 1 TeV (open circles) incident electrons. From Maire (2001).

In calorimeters with thickness ~ 25 X_0, the shower leakage beyond the end of the active detector is much less than 1% up to incident electron energies of ~ 300 GeV (LHC energies).

Quiz

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An electron propagates through a block of lead (X^0 = 5.6 mm). Compute the expected average number of particles generated in the shower after 2.8 cm.

Explain why calorimeters are better suited for the energy measurement than magnet spectrometers.