

**Problem 1**

Calculate the center-of-mass energy for fixed-target proton-proton collisions at the LHC, colliding one of the two LHC beams on a target at rest (this is actually done in LHCb, see e.g. <https://cerncourier.com/a/new-smog-on-the-horizon/>).

**Problem 2**

Estimate the expected mass resolution for the  $\Upsilon(1S)$  in LHCb. As shown in the slides, the mass of a particle decaying into a  $\mu^+$  and a  $\mu^-$  can be calculated from the measured momenta of the two muons as

$$m_{\mu^+\mu^-} = \sqrt{(E_{\mu^+} + E_{\mu^-})^2 - (\vec{p}_{\mu^+} + \vec{p}_{\mu^-})^2}$$

with

$$E_{\mu^\pm} = \sqrt{|\vec{p}_{\mu^\pm}|^2 + m_\mu^2}$$

Use the approximation  $|\vec{p}_{\mu^\pm}| \gg m_\mu$  to express  $m_{\mu^+\mu^-}$  as a function of  $|\vec{p}_{\mu^+}|$ ,  $|\vec{p}_{\mu^-}|$  and  $\cos\theta_{\mu^+\mu^-}$ , where  $\theta_{\mu^+\mu^-}$  is the opening angle between  $\vec{p}_{\mu^+}$  and  $\vec{p}_{\mu^-}$ .

Assume  $m_{\mu^+\mu^-} = 10$  GeV (i.e. approximately the mass of the  $\Upsilon(1S)$  resonance) and  $|\vec{p}_{\mu^+}| = |\vec{p}_{\mu^-}| = 50$  GeV. Calculate  $\cos\theta_{\mu^+\mu^-}$ . In the plot shown on slide 35 of the lecture you can see that the relative momentum resolution for a 50 GeV particle in LHCb is about 0.7%. Assume further that  $\theta_{\mu^+\mu^-}$  can be measured with relative precision  $3 \times 10^{-4}$ . Estimate the expected uncertainty on  $m_{\mu^+\mu^-}$  using Gaussian error propagation. Which of the two uncertainties (that on  $|\vec{p}_{\mu^\pm}|$  or that on  $\theta_{\mu^+\mu^-}$ ) dominates the mass resolution? Compare your result with the measured mass resolution for the  $\Upsilon(1S)$  of about 47 MeV (see e.g. [https://lhcb-public.web.cern.ch/Images\\_2012/Images\\_2010/UpsilonLHCb2.png](https://lhcb-public.web.cern.ch/Images_2012/Images_2010/UpsilonLHCb2.png)).