Physics of shower development

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Bibliography (figures are taken from the following references):

1. A. Lechner, Particle Interactions with Matter, CERN-2018-008-SP
2. R. Wigmans, Calorimetry: Energy Measurement in Particle Physics, OUP 2017

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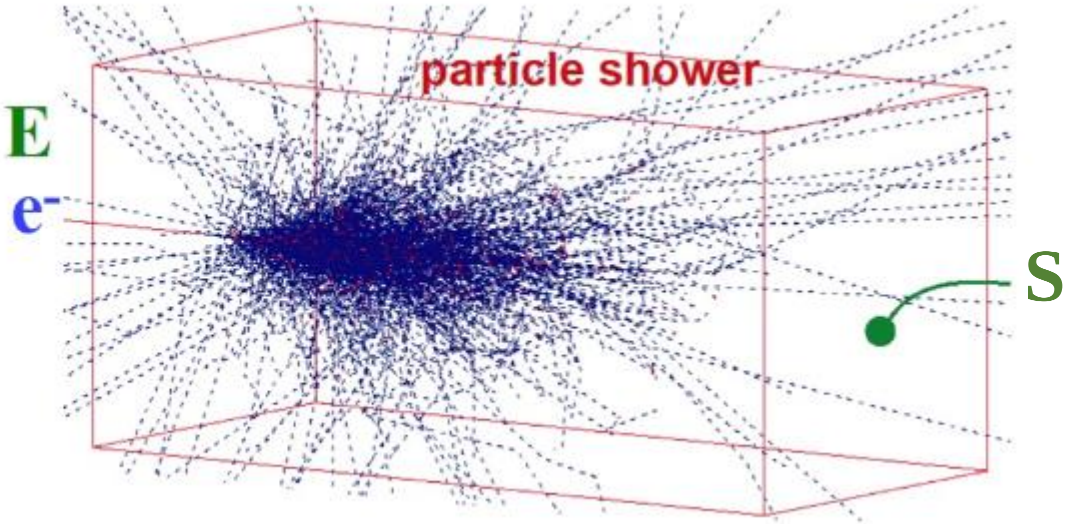
1. Introduction



- From the detector response S:
  - detect the incident particle
  - calculate its energy E and other properties

1.1 Calorimetry

- Detection of particles and measurement of their properties through absorption in a medium = "*calorimetry*"
  - in absorption, all particles' energy is converted to heat
- But, 1 calorie = 10^7 TeV (→ can't use temperature)
  - instead: measure total energy, position of deposited energy, direction and type of incoming particle
- Particles traversing matter:
  - lose energy by interacting with Coulomb field of electrons and nucleons in atoms
  - cause nuclear reactions
  - generate *showers*

1.2 Common particle properties

	Particle type	Rest mass [MeV/c]	Mean life τ [s]	Main decay mode
Photons	(γ)	0	Stable	–
Leptons	Electron (e^-), positron (e^+)	0.511	Stable	–
	Muon (μ^+, μ^-)	105.66	2.2×10^{-6}	$\mu^- \to e^-\overline{\nu}_e\nu_\mu$ $\mu^+ \to e^+\nu_e\overline{\nu}_\mu$
Hadrons	Proton (p)	938.27	Stable	–
	Neutron (n)	939.57	880	$n \to pe^-\overline{\nu}_e$
	Charged pion (π^+, π^-)	139.57	2.6×10^{-8}	$\pi^- \to \mu^-\overline{\nu}_\mu$ $\pi^+ \to \mu^+\nu_\mu$
	Neutral pion (π^0)	134.98	8.5×10^{-17}	$\pi^0 \to \gamma\gamma$

table from (1)

1.3 Cross section (σ) and mean free path (λ)

- *Cross-section* σ(E,Z,A) = (in)elastic collision probability of incident particle (energy E) with atom (Z,A)
  - Effective area for collision. Unit: barn. 1 b = 10^{-24} cm^2
- *Mean free path* λ = average distance between successive collisions: $$\lambda = \frac{1}{N\sigma} = \frac{M}{\rho N_A \sigma}$$
  - (N=atom density, M=molar mass, ρ=material density, N_A = Avogadro's number)
- Collision probability between l and l+dl: $$p(l)\mathrm{d}l = \frac{1}{\lambda}\exp\left(-\frac{l}{\lambda}\right)\mathrm{d}l$$
- *Interaction* and *survival probability* up to path l:

$$P(l) = \int_0^l p(l')\mathrm{d}l' = 1 - \exp\left(-\frac{l}{\lambda}\right)$$

$$P_s(l) = 1 - P(l) = \exp\left(-\frac{l}{\lambda}\right)$$

1.4 Example: survival of protons on a heavy target

- 400 GeV proton beam on SHiP target (54 cm Mo λ=15cm, 71 cm W λ=9cm) plus hadron absorber (500 cm Fe λ=17cm)
  - $4*10^{13}*(e^{-54/15}* e^{-71/9}* e^{-500/17}) \sim 4.6e^{-5}$
- Thus, all protons have an inelastic interaction in the target + hadron absorber
- For 5 years operation ($2*10^{20}$ protons): 345 protons survive
- But for a 250cm hadron absorber:
  - ~ 113 protons per pulse survive

1.5 Charged particle interactions

- See M. Patel’s lecture. Three types relevant for electromagnetic showers:
    1. Ionization
    2. Deflection
    3. Radiation

1.5.1 Ionization

- Coulomb interactions (=collisions) with electrons:
  - ejected electrons (mainly low energy) → measurable signal
  - sufficiently energetic to cause secondary ionization: *δ-rays*
- e^+, e^- <~10MeV, μ^+, μ^- <~100GeV, charged hadrons <~TeV
- mean energy loss per unit path length = *electronic stopping power*: $\frac{dE}{dx}\big|_{\mathrm{elec}}$
- Bethe-Bloch (for the derivation, see M. Patel's lecture; for the application to particle identification, see A. Golutvin's lecture) formula for the ionization density:

$$-\langle dE/dx \rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]$$

in which $T_{\max}$ represents the maximum kinetic energy that can be imparted to an electron in a single collision, I is the mean excitation energy of the absorber material, δ a correction term describing the *density effect*, and the proportionality constant K equals $4\pi N_A r_e^2 m_e c^2$. ref. (2)

1.5.2 Deflection

- Coulomb interactions with nuclei dominate the angular deflection
  - larger energy, smaller deflection
  - “non-ionizing” energy loss << electronic energy loss
  - $\rightarrow$ multiple scattering
  - mean energy loss per unit path length due to Coulomb collisions with nuclei = *nuclear stopping power*: $\frac{dE}{dx}\Big|_{\text{nuclear}}$

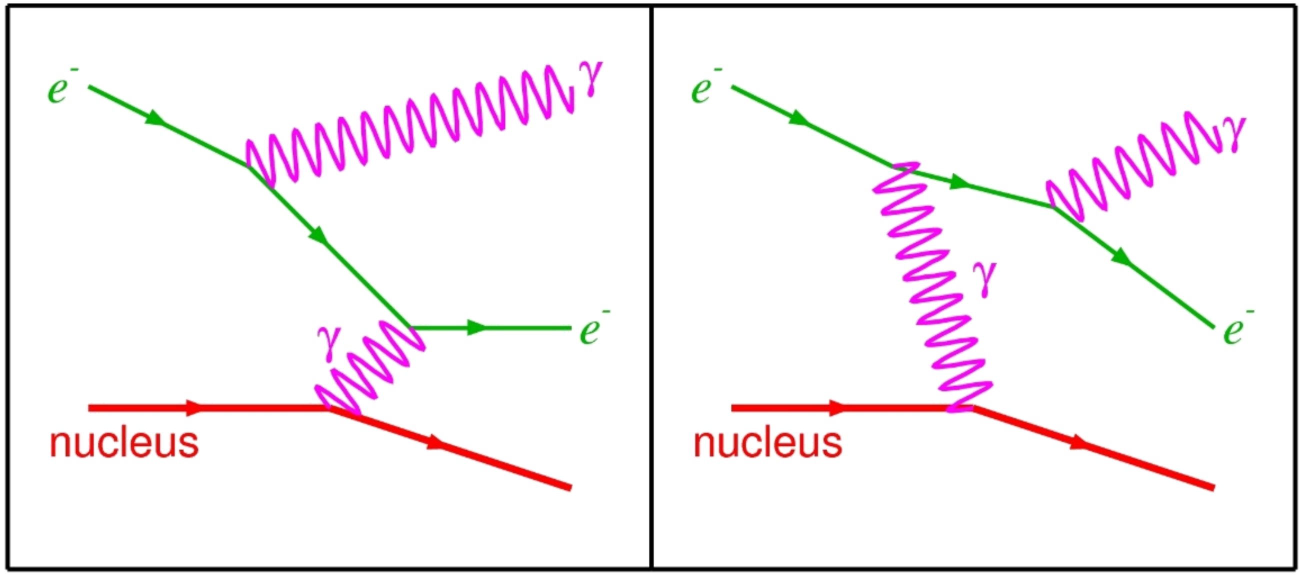
1.5.3 Radiation

- Charged particle interacts with Coulomb field of nucleus
- Electrons > 10 MeV (=E_c , critical energy) lose energy via a Bremsstrahlung photon
- *Critical energy* = energy E_c where *electronic* and *radiative stopping power* are equal:

  $$\frac{dE}{dx}\Big|_{\text{elec}}(E_c) = \frac{dE}{dx}\Big|_{\text{rad}}(E_c)$$

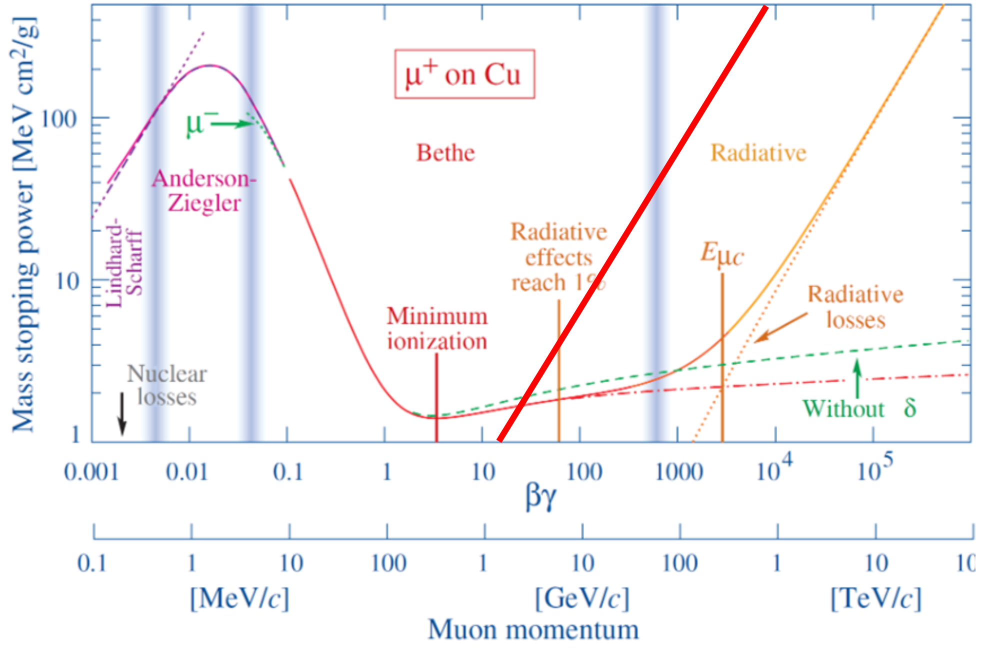
- Critical energy ~$(m_p/m_e)^2$: electron in copper E_{ec}= 20 MeV, muon in copper $E_{\mu c}$= 400 GeV
- E_c ~ 610 MeV/(Z+1.24)

Bremsstrahlung



1.5.4 Mass stopping power

- *mass stopping power* = material density / stopping power (units: MeV cm^2/g)
- Minimum of Bethe at βγ ~ 3-3.5. Charged particles in this regime: *minimum ionizing particles* (*MIPS*). Their mass stopping power 1-2 MeV cm^2/g



Electron momentum 5 50 500 MeV/c

figure from (2)

1.5.5 Radiation length X_0

- At high energies: $\frac{dE}{dx}|_{rad} \sim E$ (slide 12), hence: $-\frac{dE}{dx} = \frac{E}{X_0}$
- Inverse constant of proportionality X_0 = *radiation length*
  - Average energy of high energy electrons/positrons decreases exponentially with traversed length: $<E(x)> = E_0 e^{-x/X_0}$ (E_0= energy at $x = 0$)
  - X_0 = distance over which 1-1/e (=63.2%) of energy lost due to Bremsstrahlung. e.g. X_0 (Pb) = 5.3 mm.
- Tsai's formula for radiation lengths:

$$X_0' = \frac{716.4 \text{ g/cm}^2 A}{Z(Z+1)\ln(287/\sqrt{Z})},$$

where X_0' is $X_0\rho$ in g/cm^2, and Z and A are the atomic and mass numbers, respectively.

ref. (2)

- $X_0 \sim A/Z^2\rho$

1.6 Photon interactions

Three important types:

1. Photo electric effect
   - at low energies, atom absorbs photon and emits electron
   - $\sigma \sim Z^{4\text{-}5}$ and $\sigma \sim E_\gamma^{-3}$
   - in uranium (Z=92) dominating effect < 700 keV, iron (Z=26) < 100 keV, carbon (Z=6) < 20 keV
2. Compton scattering
3. Pair production

1.6.1 Compton scattering

- Photon scattered by atomic electron with momentum and energy transfer to scattered electron (now unbound)
- $\sigma \sim Z$, $\sigma \sim E_\gamma^{-1}$
- dominant effect between few 100 keV and 5 MeV
- γs in MeV range absorbed in a sequence of Compton scattering processes until E_γ reduced to photo electric absorption
- at least half of total energy deposited by such γs in absorption of e^+, e^- or γ ⟶ Compton scattering important process for calorimetry

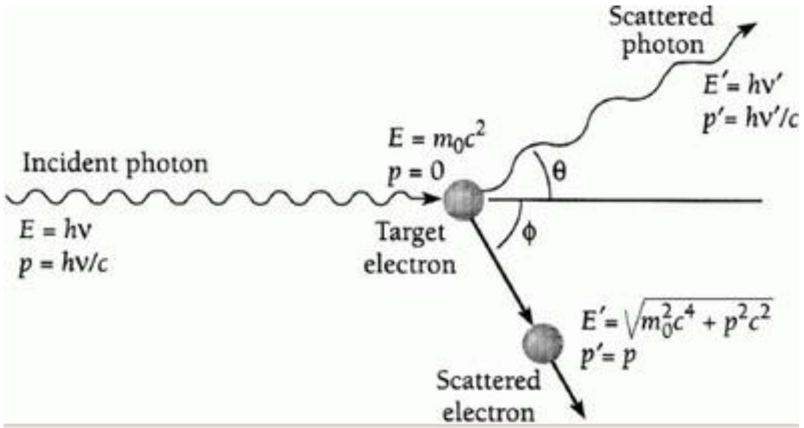
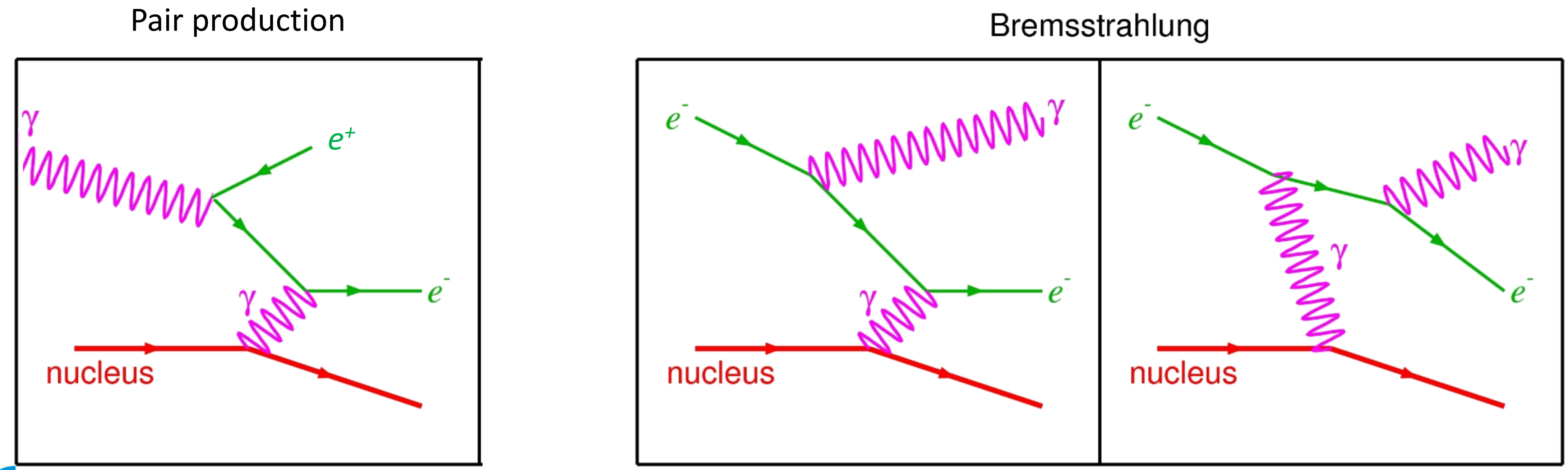


Fig. 2.4. The Compton scattering process.

figure from (2)

1.6.2 Pair production

- For $E_\gamma > 2 * m_e$, pair production can occur in the Coulomb field of the nucleus
- $\sigma \sim Z^2$, $\sigma \sim \ln(E_\gamma)$
- Inverted Bremsstrahlung (see slide 11):



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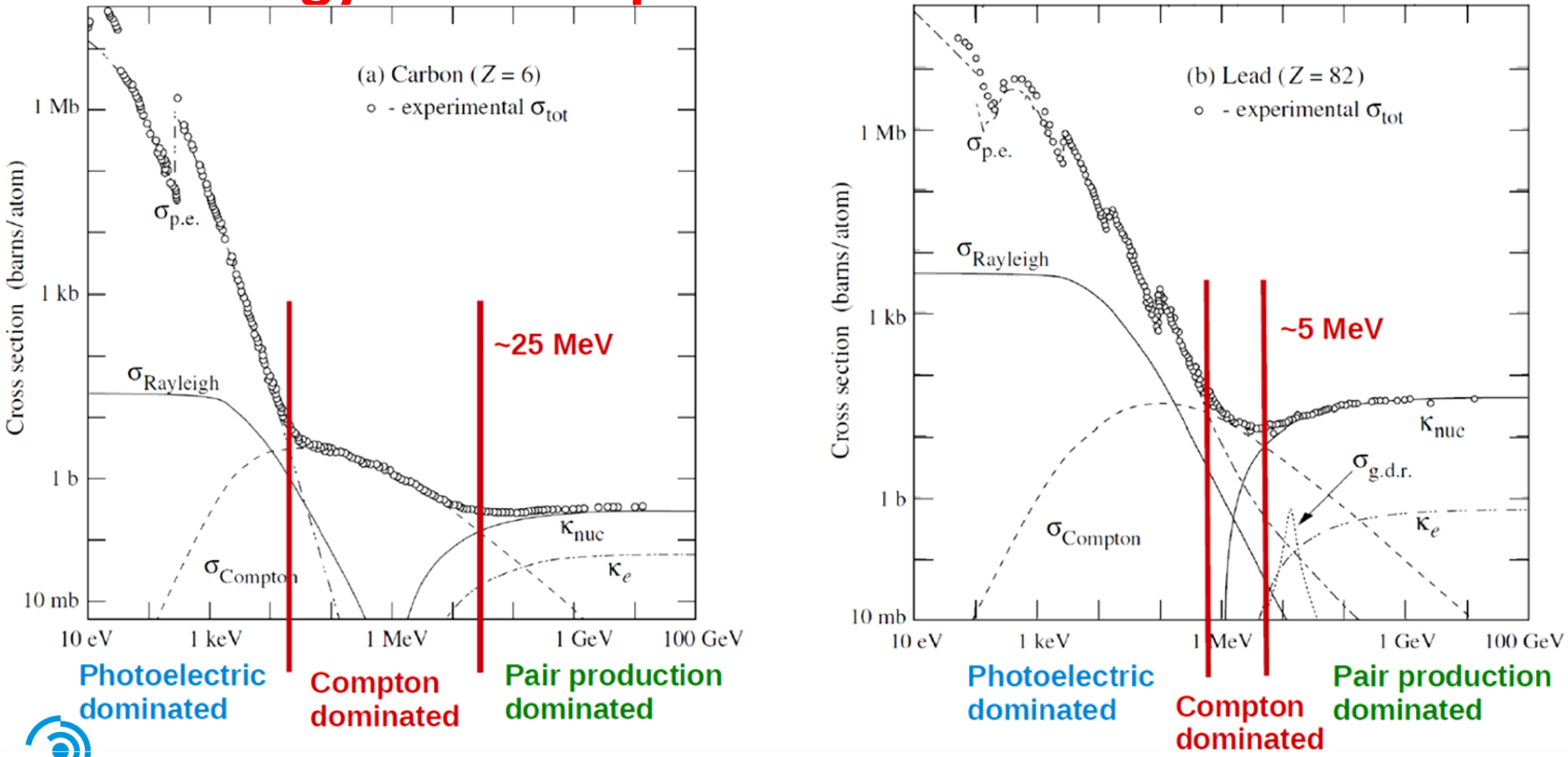
1.6.3 Energy scales of photon interactions



1.6.4 Radiation length and mean free path

- Radiation length (X_0) also characteristic length for photon penetration in matter. Pair production cross section and mean free path (λ) constant at high energies (see slide 17) :

$$\sigma_{\mathrm{pair}} \approx \frac{7}{9}\frac{M}{\rho N_{\mathrm{A}} X_0}, \qquad \lambda_{\mathrm{pair}} \approx \frac{9}{7} X_0$$

- photon survival probability I(x) for the photon intensity at depth x (μ= *attenuation coefficient*):

$$I(x) = I_0 \cdot \exp\left(-\frac{7}{9}\frac{x}{X_0}\right) = I_0 \,.\, \exp(-\mu x) \ , \mu = \frac{7}{9X_0}$$

- X_0 is a characteristic length for Bremsstrahlung and pair production:
  - central quantity for describing longitudinal development of showers

1.7 Difference photons and electrons

- Electrons lose energy in continuous stream of events in which atoms are ionized and bremsstrahlung photons are emitted
- Photons may penetrate considerable amount of matter without losing any energy, then interact in a manner that changes their identity (i.e. a photon may turn into an e^+e^- pair)
- A photon traversing 1cm lead has 75% probability of interacting (mean free path of 7.2 mm) but an electron will lose 83% of its energy

1.8 Quiz

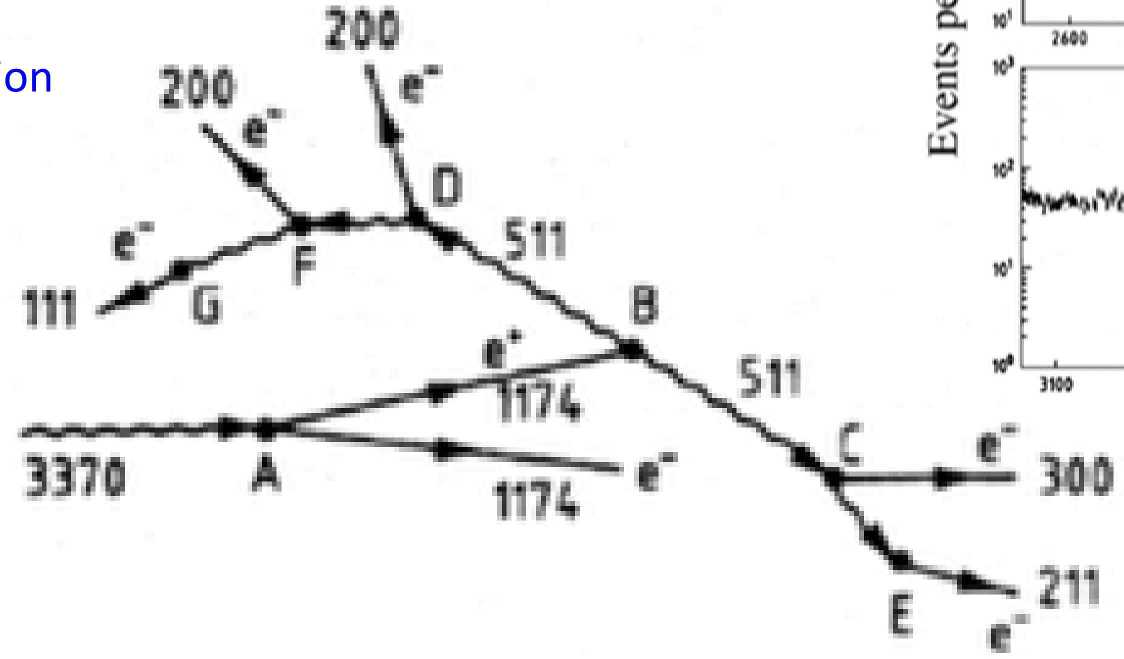
By now you should know:

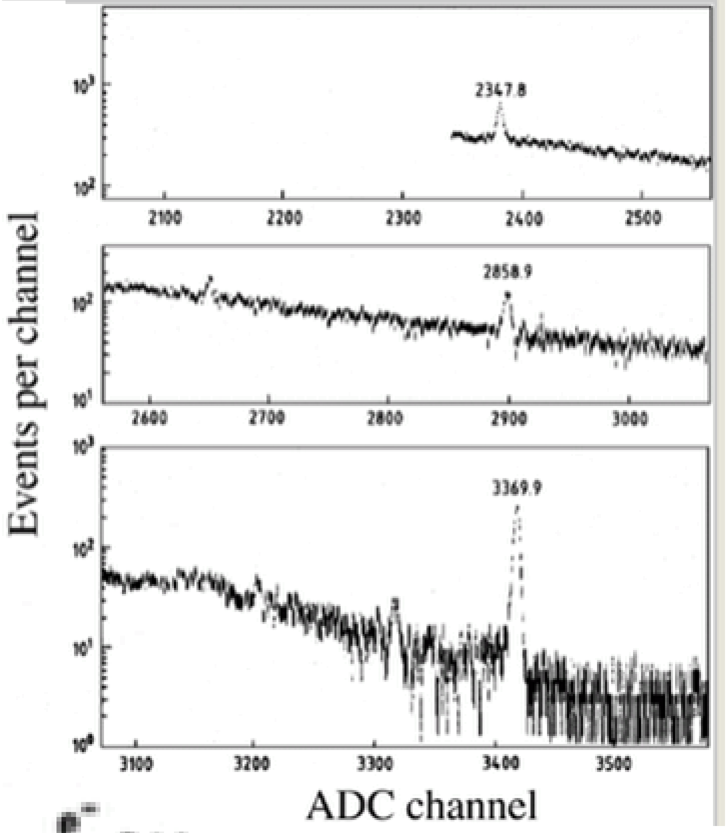
- what a cross section is and how it relates to the mean free path
- three types of charged particle interactions in matter
- three types of photon interactions with matter
- what the critical energy is
- what the radiation length is and how it is related to the mean free path
- the difference between the interactions of photons and electrons

If you are unsure, go back to the slides.

2. Electromagnetic showers: low energy example

- γs of 3370 keV, produced by the decay of ^{65}Ga:
  - nuclear de-excitation photons
  - A: e^+ /e^- pair production
  - B: e^+ at rest annihilates, creating 2 γs of 511 keV
  - C,D,F: Compton scattering
  - E,G: photo electric absorption
- energy deposited by 1 positron and 6 electrons through ionization





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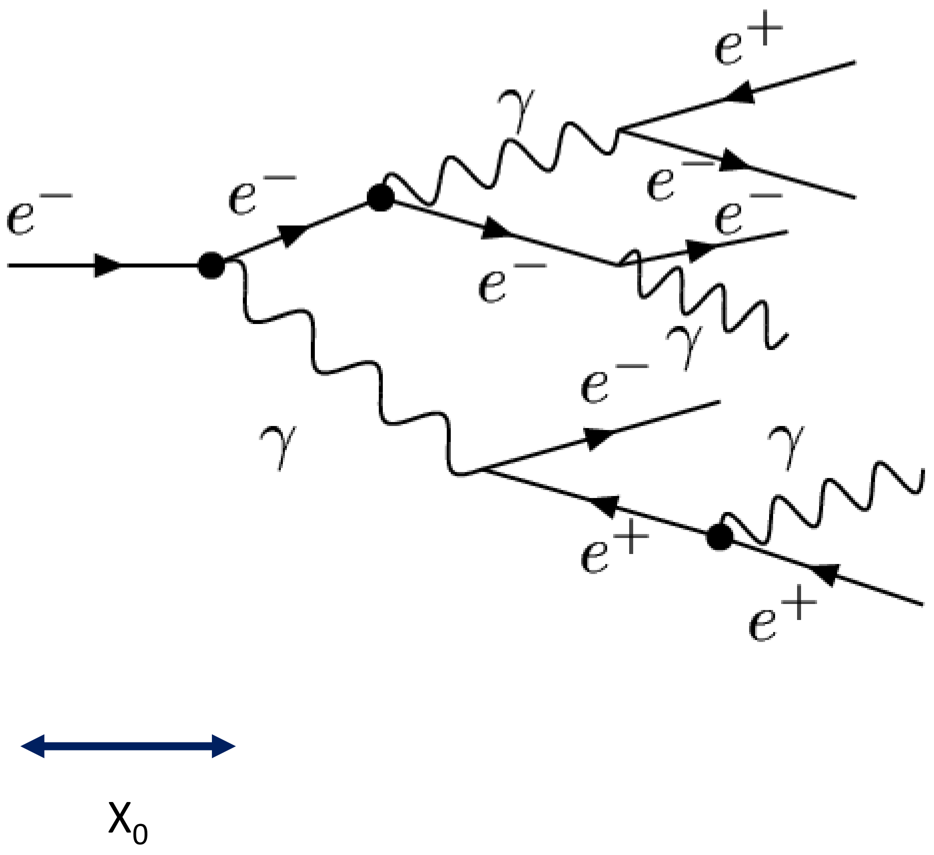
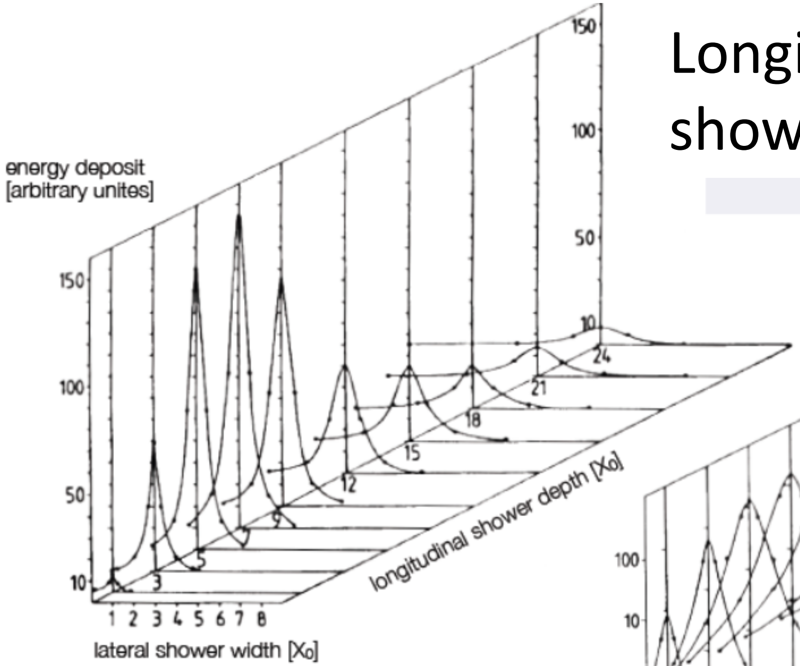
2.1 Bremsstrahlung + Pair Production

- High energies: *Bremsstrahlung* main source of energy loss by electrons and positrons
- *Multiplication* of particles:
  - primary multi-GeV e^- radiates thousands of photons
  - photons are soft and absorbed by Compton scattering and photo electric effect
  - photons with E > 5-10 MeV can create e^+/e^- pairs
  - fast e^+, e^- radiate more photons which create more pairs
- Energy is lost in a *multi-step* process:
  - → *Shower* containing thousands of electrons, positrons and photons

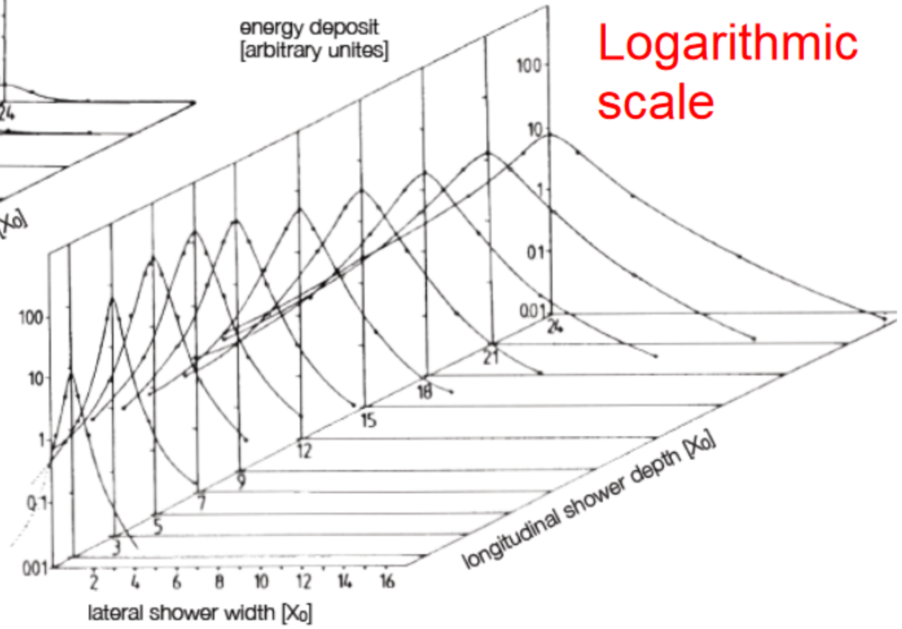


2.2 Shower development

Longitudinal and transverse EM shower profile of 6 GeV e^- in lead



Linear scale



Logarithmic scale

2.2.1 Development process

- Shower energy deposited by:
  - ionizing electrons and positrons
  - electrons created in Compton scattering and photo electric processes
  - majority of shower particles that deposit energy are soft (Compton & photo electric)
  - ¼ by e^+ , ¾ by e^-
- Initially, deposited energy increases with increasing shower depth
- As shower develops, particles become softer until no further multiplication can take place:
  - *shower maximum,* beyond which:
    - photons produce only one electron (Compton or photo electric)
    - number of particles and deposited energy decreases
  - relative importance of Compton, photoelectric and pair production changes
    - relationship between deposited energy and signal is a function of the depth in sampling calorimeters

2.2.2 Development (scaling variables X_0 and ρ_M)

- Material independent: radiation length X_0 (longitudinal) and *Molière radius* ρ_M (transverse)
  - X_0(Al) = 89 mm, X_0(Fe) = 17.6 mm, X_0(Pb) = 5.6 mm ($X_0 \sim A/Z^2\rho$)
  - Increasing Z:
    - shower max at greater depth (in nb of X_0)
    - shower decays more slowly (in nb of X_0)
    - more X_0 required to contain the shower

Lateral spread:

- central part: multiple Coulomb scattering of e^+, e^-
- tail: low energy photons and electrons from (isotropic) Compton scattering and photoelectric effect
- ρ_M = average lateral deflection of electrons with $E=E_c$ after traversing one radiation length X_0

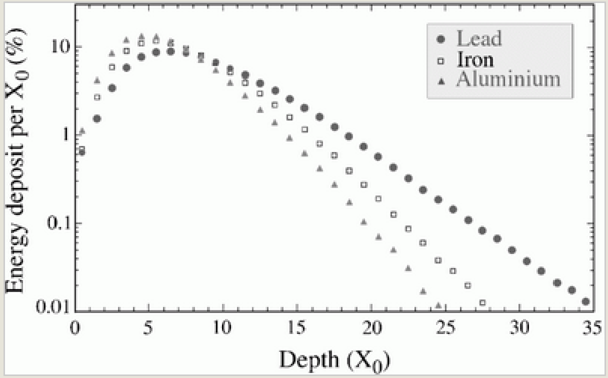


Fig. 2.12. Energy deposit as a function of depth, for 10 GeV electron showers developing in aluminium, iron and lead, showing approximate scaling of the longitudinal shower profile, when expressed in units of radiation length, X_0.

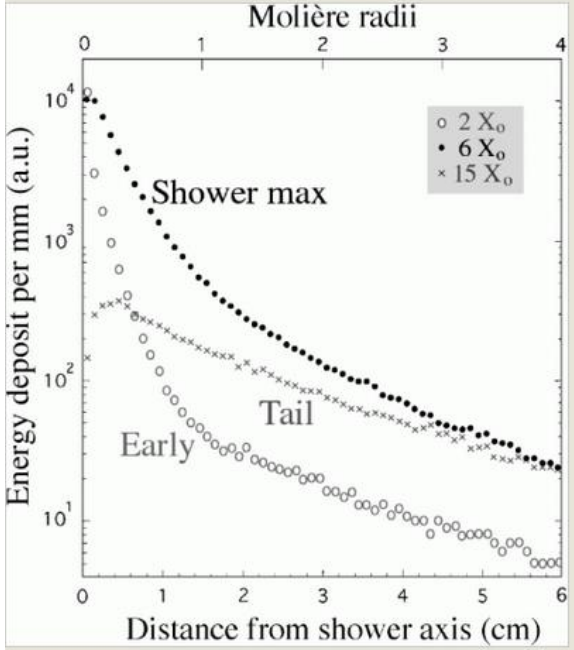


Fig. 2.13. The radial distributions of the energy deposited by 10 GeV electron showers in copper, at various depths.

figures from (2)

2.2.3 *Longitudinal profile*

- Heitler model: Bremsstrahlung and pair production take place after distance X_0
  - energy split between outgoing particles
- nb particles after *t* radiation lengths: $N(t) = 2^t$, energy per particle $E(t)=E_0/2^t$ (E_0 incident particle energy)
- cascade stops when $E=E_c$
- *penetration depth (shower maximum):* $t_{max} \sim \log(E_0/E_c)$
- $N_{max} = E_0/E_c$
- proportional to initial energy

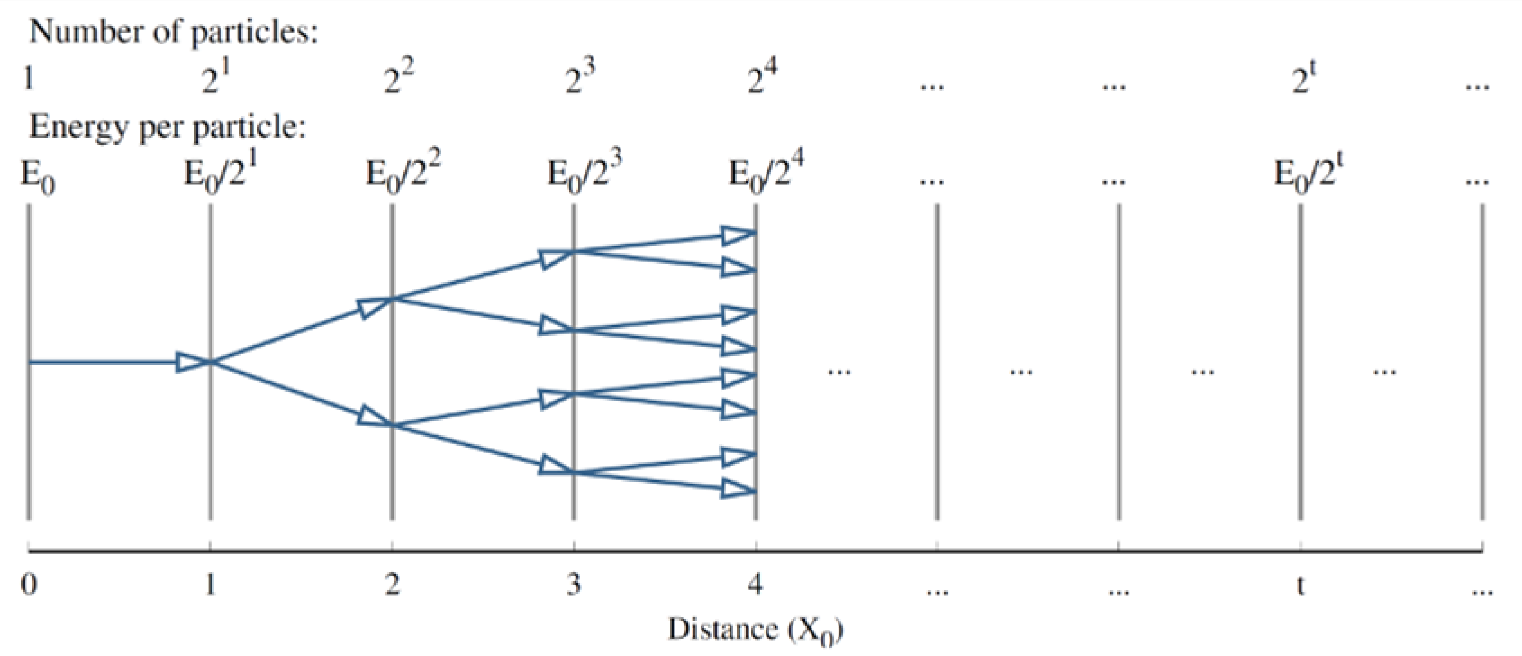


figure from (1)

2.2.4 *Transverse profile*

- Molière radius ρ_M : average lateral deflection of electrons with $E=E_c$ after traversing one radiation length X_0
- $\rho_M =$ $$\frac{\sqrt{4\pi\alpha} m_e c^2}{E_c} X_0 \approx \frac{21\text{ MeV}}{E_c} X_0$$

  where α is the fine structure constant, and $m_e c^2$ is the electron mass.
- >90% of beam energy deposited within a cylinder with radius ρ_M around the beam axis
  - 95% : $2\rho_M$
- Lateral width increases with longitudinal shower depth
- Molière radius Z independent (remember: $X_0 \sim A/Z^2\rho$ and $E_c \sim 610\text{ MeV}/(Z+1.24)$)

2.3 Containment

- Important for calorimeter design (see G. De Lellis lecture Module 3, lecture 3)
- Longitudinal containment determines depth
  - 99% of 100 GeV e^- shower: 25 X_0 (Pb, 25*5.6=140mm), 22 X_0 (Cu, 22*14.3=315mm)
  - depth to contain showers increases with log E
- Lateral containment determines cell size (*granularity,* overlapping showers!)
  - approximatively Z independent
  - ρ_M(Pb)=16mm, ρ_M(Cu)=15.2mm
- 95% of energy within:

  $L(95\%)=t_{max}+0.08\ Z + 9.6\ X_0$

  $R(95\%)=2\ \rho_M$

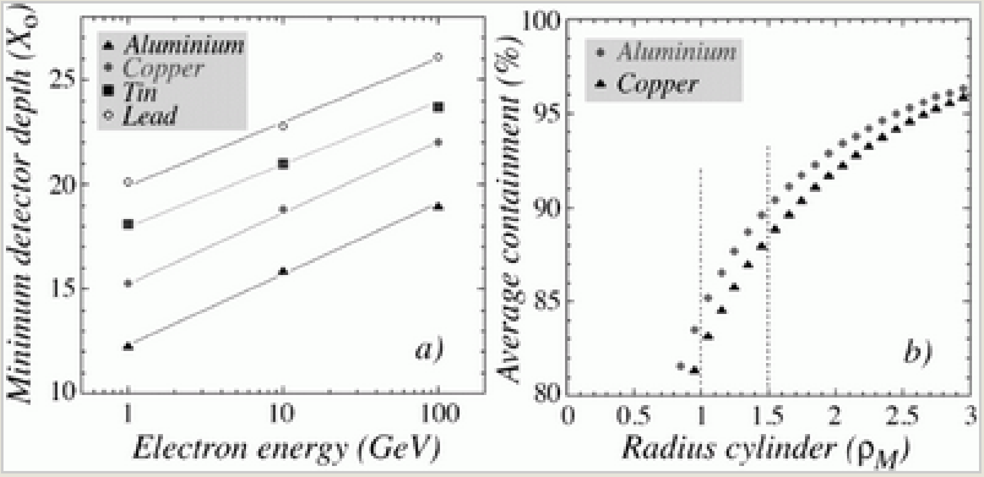
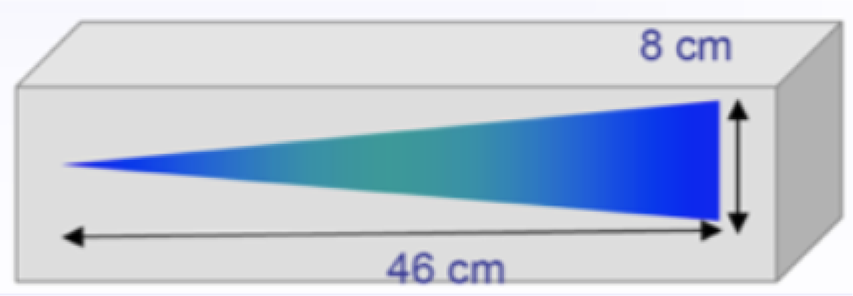


Fig. 2.19. Size requirements for electromagnetic shower containment. The depth of a calorimeter needed to contain electron showers, on average, at the 99% level, as a function of the electron energy. Results are given for four different absorber media (*a*). Average energy containment of electron-induced showers in a copper and an aluminium based calorimeter, as a function of the radius of an infinitely deep cylinder around the shower axis (*b*).

figure from (2)

2.3.1 Containment: example

- Electromagnetic showers in lead glass (OPAL detector at LEP)
  - $X_0 \approx 2$ cm, $E_C = 11.8$ MeV, $\rho_M = 1.8\ X_0 \approx 3.6$ cm
  - For $E_0 = 100$ GeV one obtains: $t_{max} \approx 13$
  - Z(Pb) = 82
  - Longitudinal containment: $L(95\%) \approx (13 + 0.08\ Z + 9.6\ X_0) = 39$ cm
  - Lateral containment: $R(95\%) = 2\ \rho_M = 7.2$ cm
- Electromagnetic calorimeters are compact



2.4 Energy resolution

- fluctuations in energy measurement (leakage)
- assume particle with energy E creates a signal S consisting of on average n signal quanta
  - event to event fluctuations in the signal correspond to Poissonian fluctuations in n
  - relative width of signal distribution, $\sigma_S/\langle S \rangle$, i.e. relative precision of energy measurement σ_E/E:
    - $\sqrt{n}/n = 1/\sqrt{n}$
  - In practice, energy resolution of a calorimeter has various uncorrelated contributions:

$$\sigma_E = a\sqrt{E} \oplus bE \oplus c$$

  - a stochastic term, b constant term, c noise term

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

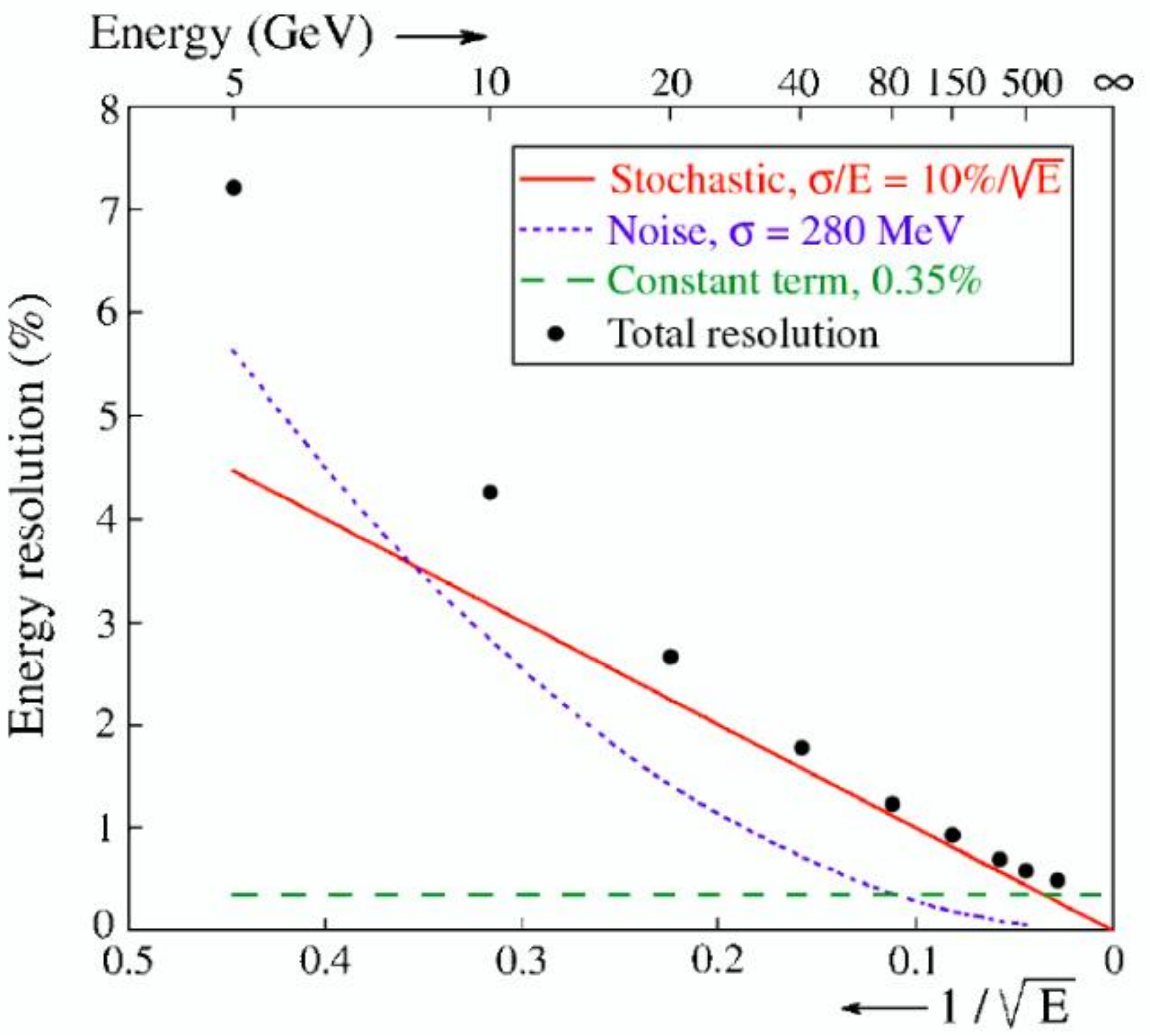
N.B. For a measurement depending on a and b with uncorrelated uncertainties, their uncertainties are added in quadrature:

$$\delta(a+b) = \sqrt{\delta a^2 + \delta b^2} =: \delta a \oplus \delta b$$

2.4.1 Energy resolution: example

ATLAS EM calorimeter

- Contributions to the energy resolution σ_E/E:
  - Shower fluctuations, stochastic term ~ $1/\sqrt{E}$
  - Electronic noise ~ $1/E$
  - Leakage, calibration ≃ constant



2.5 Muons

- up to ~100 GeV, primarily lose energy via ionization and δ-rays (Bethe Bloch)
  - mass stopping power: ~1–2 MeV $g^{-1}cm^2$, stopping power in iron ~ 1.1 GeV/m
  - need a lot of material to absorb muons (CERN's WANF 300 m long iron shield)
- at higher energies (iron $E_{\mu c}$= 400 GeV), radiative losses become dominant
  - material dependent, increase w.r.t ionization loss factor of 2-6

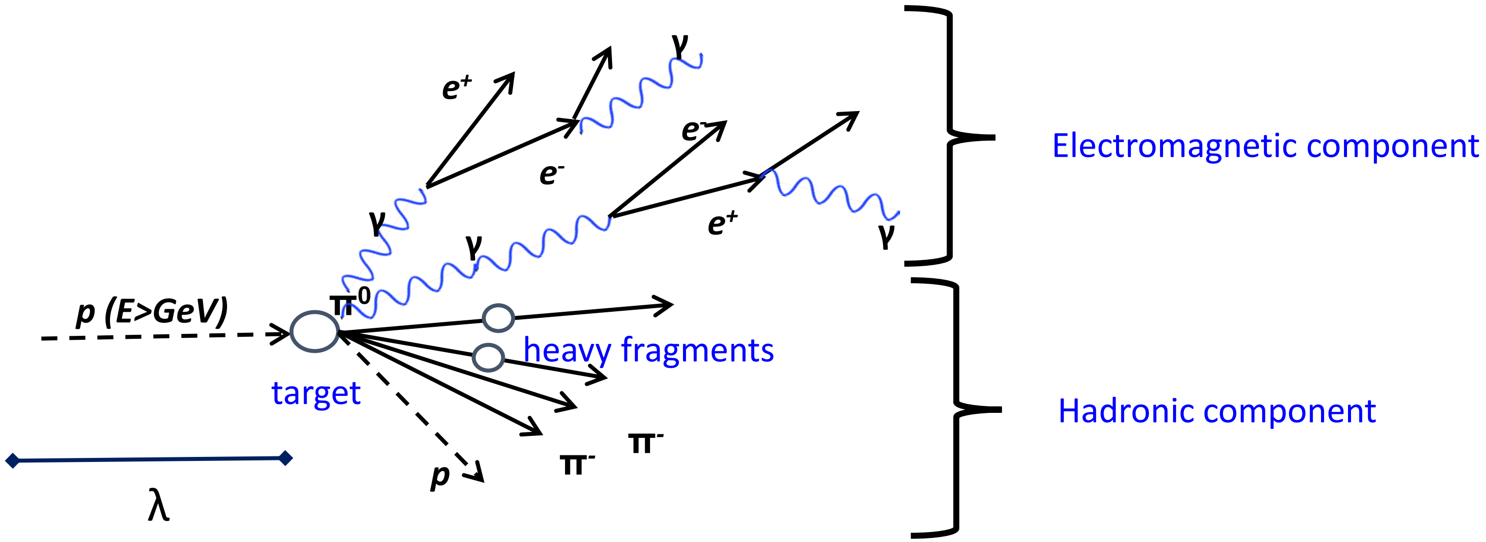
2.5 Quiz

By now you should know:

- the two processes that cause electromagnetic showers
- how an incident particle looses its energy in an absorber
- the scaling variables used to describe lateral and transverse shower development
- the Heitler model to calculate the energy dependence of the shower depth
- criteria for shower containment
- how to calculate the energy resolution of electromagnetic showers

If you are unsure, go back to the slides.

3. Hadronic showers



- In nuclear collisions secondary particles are produced
- Particles can decay electromagnetically (π,η): electromagnetic showers
- But can also undergo secondary, tertiary nuclear interactions: hadronic cascade

3.1 Difference between electromagnetic and hadronic showers

- Shower development similar to electromagnetic showers, but:
- Hadronic showers are more complex
- Hadronic showers governed by strong rather than electromagnetic interactions
  - larger variety of processes
  - fraction of dissipated energy (to break apart nuclei) is *undetectable*
- Scales of hadronic and electromagnetic showers determined by differences between cross sections for nuclear and electromagnetic reactions

3.2 Nuclear interaction length (λ_{int})

- *nuclear interaction length* λ_{int} of an absorber = average distance a high energy hadron has to travel before a nuclear interaction occurs
- probability traversing distance z without nuclear interaction: $P = \exp(-z/\lambda_{int})$
- cross section for nuclear interactions: $\sigma_{tot} = \frac{A}{N_A \lambda_{int}}$, $\sigma \sim r^2$, $r^3 \sim A \rightarrow \lambda_{int} \sim A^{1/3}$ (g/cm^{-2})
- 1 λ_{int} ~ 10 cm for heavy materials (W, U, see next slide)
- $\sigma_{\pi p}$(24 mb)< σ_{pp} (38 mb) (at 100 GeV) $\rightarrow$ λ_{int} (π) 25% greater than λ_{int} (p) because π's are smaller
  - π travel 25-50% longer than p and the shower starts later
  - baryon number conservation: leading particle in the interaction is also a baryon with shorter mean free path

3.2.1 Material properties (λ_{int} for p)

- $X_0 \sim \frac{A}{\rho Z^2}$, $\lambda_{int} \sim \frac{A^{1/3}}{\rho}$
- $\frac{\lambda_{int}}{X_0} \sim A^{4/3}$
- $\lambda_{int} >> X_0$
- $\rightarrow$ need more material to contain hadronic showers

Passive material	Z	Density (g cm^{-3})	ϵ_c (MeV)	X_0 (mm)	ρ_M (mm)	λ_{int} (mm)	$(dE/dx)_{mip}$ (MeV cm^{-1})
C	6	2.27	83	188	48	381	3.95
Al	13	2.70	43	89	44	390	4.36
Fe	26	7.87	22	17.6	16.9	168	11.4
Cu	29	8.96	20	14.3	15.2	151	12.6
Sn	50	7.31	12	12.1	21.6	223	9.24
W	74	19.3	8.0	3.5	9.3	96	22.1
Pb	82	11.3	7.4	5.6	16.0	170	12.7
^{238}U	92	18.95	6.8	3.2	10.0	105	20.5
Concrete	-	2.5	55	107	41	400	4.28
Glass	-	2.23	51	127	53	438	3.78
Marble	-	2.93	56	96	36	362	4.77

table from (2)

3.3 Longitudinal profiles

- 8 λ_{int} Uranium (85 cm) to contain 95% of the hadronic showers of 300 GeV π^- beam
- 300 GeV electrons would require 10cm of Uranium
- absorption of hadron showers requires much more material than em showers of the same energy

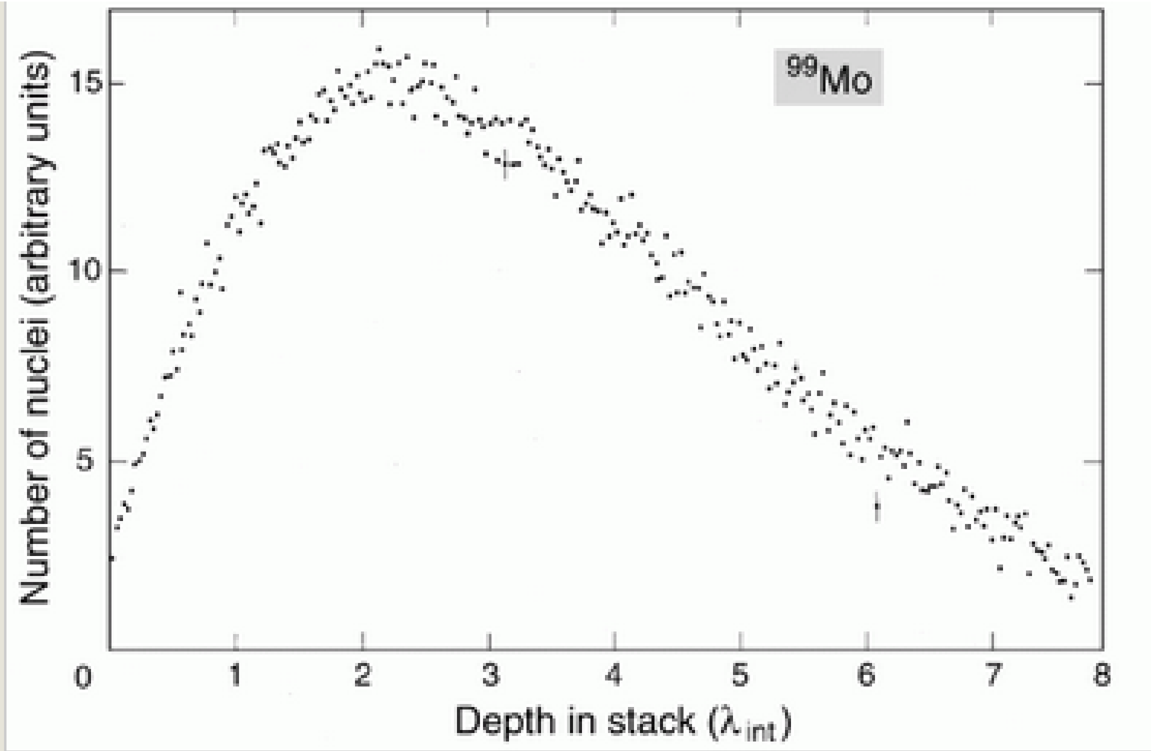


Fig. 2.34. Longitudinal shower profile for 300 GeV π^- interactions in a block of uranium, measured from the induced radioactivity. The ordinate indicates the number of radioactive decays of a particular nuclide, ^{99}Mo, produced in the absorption of the high-energy pions.

figure from (2)

3.4 Lateral profiles

- narrow core: em shower from π^0 s
- exponentially decreasing halo from non em component
- hadron showers are broader than em showers
  - cylinder with 32cm (1.5 λ_{int}) radius contains 95% of showers
  - nine times larger than 3.5cm (1.8 ρ_M) for 80 GeV em showers

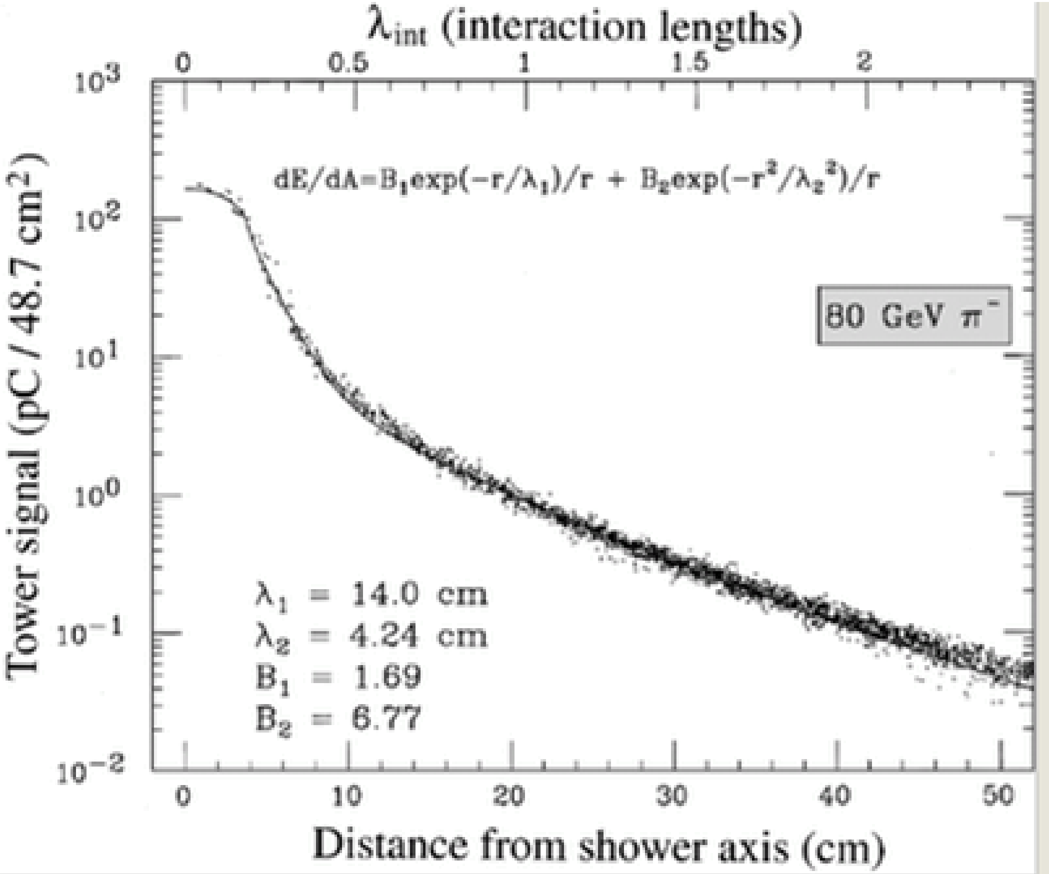


Fig. 2.38. Average lateral profile of the energy deposited by 80 GeV π^- showering in the SPACAL detector. The collected light per unit volume is plotted as a function of the radial distance to the impact point.

figure from (2)

3.5 Electromagnetically decaying particles

- hadronic showers produce π^0 and η
- $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$: propagates electromagnetically
- ~ 1/3 of mesons produced in first interaction are π^0s
  - continues in subsequent interactions if enough energy available
  - average fraction of π^0 increases with initial energy
- after each collision:
  - (1-1/3) of remaining energy available for the next generation of collisions
- average fraction of hadron energy carried by π^0s : $f_{em} = 1 - \left(1 - \frac{1}{3}\right)^n$, n ~ $E_{\text{shower initiator}}$
- reality more complicated, several effects ignored here

3.5.1 Electromagnetic fraction of showers

- em fraction of hadronic shower: $$f_{em} = 1 - \left(\frac{E}{E_0}\right)^{(k-1)}$$ increases with energy
- E_0 average energy to produce one pion, k-1 related to average multiplicity <m> (~5) and average fraction of π^0 production:

$$1 - f_{\pi^0} = \langle m \rangle^{(k-1)} \rightarrow k = 1 + \frac{\ln(1 - f_{\pi^0})}{\ln\langle m \rangle}$$

- Simulation and experiments:
  - E_0 = 0.7 GeV, k= 0.82 (Cu)
  - E_0 = 1.3 GeV, k= 0.82 (Pb)
  - Z dependence: ionisation and transfer of energy to nuclei
- f_{em}(p) 15% smaller than $f_{em}(\pi)$
  - baryon conservation: less energy for π^0 production

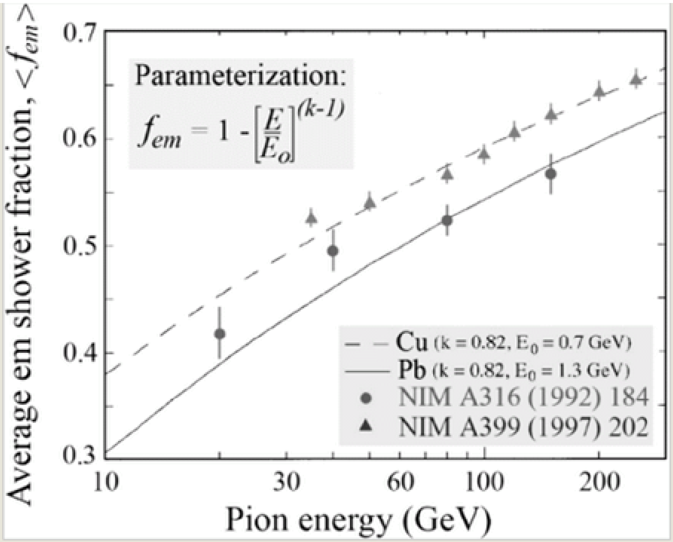


Fig. 2.25. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors.

figure from (2)

3.5.3 Ionization losses, a Z dependent effect

- Charged secondary hadrons
  - induce a nuclear interaction after having travelled one nuclear interaction length (λ_{int})
  - lose energy by ionization
- protons: 150→225 MeV
- π travel 25% more, lose 25% more: 200→300 MeV

Table 2.2 The specific ionization energy loss of minimum ionizing particles in various absorber materials, and the average energy lost by minimum ionizing protons over a distance of one nuclear interaction length. Data from [PDG 14].

Absorber	Z	dE/dx (mip) ($MeV\ g^{-1}cm^2$)	λ_{int} ($g\ cm^{-2}$)	$\Delta E/\lambda_{int}$ (MeV)
Carbon	6	1.742	85.8	149
Aluminum	13	1.615	107.2	173
Iron	26	1.451	132.1	192
Copper	29	1.403	137.3	193
Tin	50	1.263	166.7	211
Tungsten	74	1.145	191.9	220
Lead	82	1.122	199.6	224
Uranium	92	1.081	209.0	226

table from (2)

3.5.4 Particle multiplicities (example)

- 100 GeV πs on Cu (slide 41): f_{em} = 60%
- E_0(Cu) = 0.7 GeV, 40/0.7=58 hadrons (πs) in non em component
  - nb of π^0 : $1/(f(\pi^0)-1)$ ~ 1/2-1/3 of total nb of hadrons
- each hadron loses ~250 MeV (ionization) = ~15 GeV, 15% of initial energy, ~35% of non em energy
- ~65% of non em energy used to excite and dissociate atomic nuclei in the absorber
- estimate number of generations:
  - $E/E_0 = \langle m\rangle^n$, for $\langle m\rangle=5 \rightarrow n=3$
- energy dependence of f_{em} → limitations on hadronic energy resolution

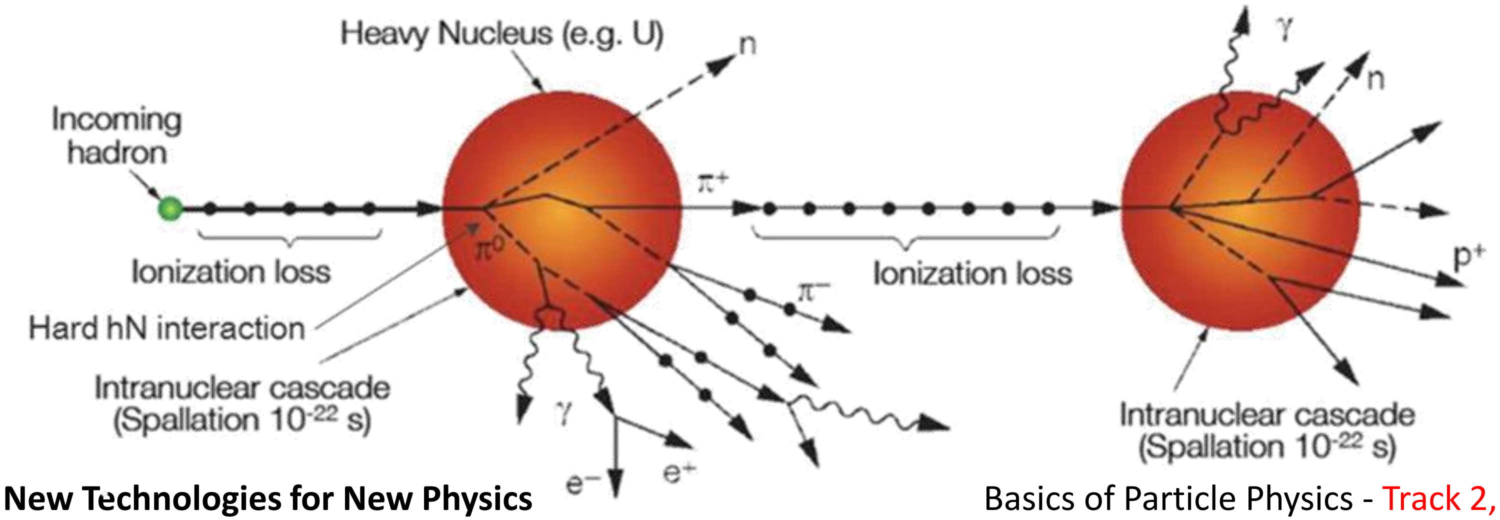
Table 2.3 Characteristics of particle production in pion-induced showers in copper (lead), calculated on the basis of Equation 2.19, with the following parameter choices: $E_0 = 0.7$ GeV (copper), 1.3 GeV (lead), $k = 0.82$, $f_{\pi^0} = 1/4$.

E_π (GeV)	$\langle f_{em} \rangle$	$\langle \#\pi^\pm, K\ldots \rangle$	$\langle \#\pi^0 \rangle$
10	0.380 (0.307)	9 (5)	3 (2)
20	0.453 (0.389)	16 (9)	5 (3)
30	0.492 (0.432)	22 (13)	7 (4)
50	0.536 (0.482)	33 (20)	11 (7)
80	0.574 (0.524)	49 (29)	16 (10)
100	0.591 (0.542)	58 (35)	19 (12)
150	0.619 (0.575)	82 (49)	27 (16)
200	0.639 (0.596)	103 (62)	34 (21)
300	0.664 (0.624)	144 (87)	48 (29)
400	0.681 (0.643)	182 (110)	61 (37)
500	0.694 (0.657)	219 (132)	73 (44)
700	0.712 (0.678)	288 (173)	96 (58)
1000	0.730 (0.698)	386 (232)	129 (77)

table from (2)

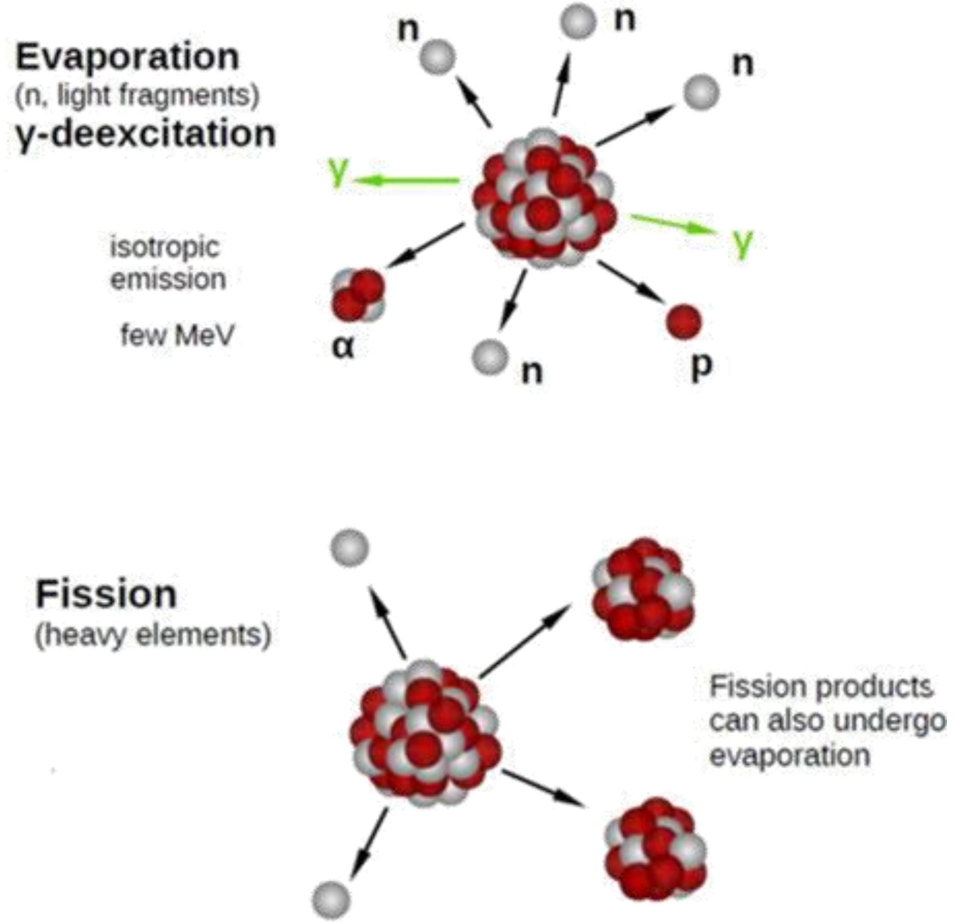
3.6.1 Nuclear sector: *spallation interactions* (1st "fast" phase)

- *Spallation* occurs when a high energy hadron hits an atomic nucleus
  - Ejection of material (spall) from a target during impact by a projectile
- Fast (10^{-22} s) intranuclear cascade, followed by a slower (10^{-16} s) evaporation stage:
  - Quasi free nucleons collide with other nucleons.
  - Pions and other hadrons may be created.
  - Some high energy particles may escape the nucleus.
  - Rest distribute kinetic energy among remaining nucleons.



3.6.2 Spallation (2nd “slow” phase)

- De-excitation of intermediate nucleus
  - *evaporation* of free nucleons, αs or heavier nucleon aggregates until excitation energy < binding energy of one nucleon
  - energy needed to release nucleons (nuclear binding energy) is lost for calorimetric purposes
    - 30-40% of non em shower energy
  - large event to event fluctuations in visible energy
    - energy resolution of hadron calorimeters worse than em energy resolution



3.6.3 Properties of spallation nucleons

- *Cascade* particles momentum component along direction of incoming particle
  - Residual target nucleus undergoes recoil
    - Recoil (kinetic energy) m/M (m=total mass of cascade nucleons, M=mass of residual nucleus)
    - Not measurable: part of invisible component of shower energy
- *Evaporation* neutrons emitted isotropically

3.6.4 Spallation: Rudstam's empiric formula

- Estimate average nb of protons and neutrons
- Estimate invisible energy
- Separate cascade nucleons from evaporation ones
- Rudstam: of atomic mass ($A > 20$), which gives a satisfactory description of spallation cross sections. When a particle of energy E hits a target with atomic mass A_{T}, the relative cross sections σ for the production of spallation products (Z_f, A_f) are given by the relation

(2.24)

$$\sigma(Z_f, A_f) \sim \exp\left[-P(A_{\mathrm{T}} - A_f)\right] \times \exp\left[-R\left|Z_f - SA_f + TA_f^2\right|^{3/2}\right]$$

in which E is expressed in MeV and the parameters P, R, S and T have the following values:

$P = 20E^{-0.77}$ for $E < 2100$ MeV, $P = 0.056$ for $E > 2100$ MeV, $R = 11.8A_f^{-0.45}$, $S = 0.486$, $T = 0.00038$.

ref (2)

3.6.5 Spallation nucleons: 1.3 GeV π on $^{208}_{82}\mathrm{Pb}$

- E_0 = 1.3 GeV (slide 41), average π loses 300 MeV (ionization) before interacting with a nucleus.
- Remaining 1.0 GeV (includes π rest mass) for excitation and dissociation of one or more nuclei.
- Pb: = (208- A_f) nucleons , (82- Z_f) protons, rest neutrons
- Compute (Rudstam) probabilities for all reactions that are energetically possible:
  - probabilities of relative nb of protons & neutrons
- 1000 MeV π s: average 2.7 protons, 12.8 neutrons
  - 16 spallation nucleons
  - much less protons than neutrons

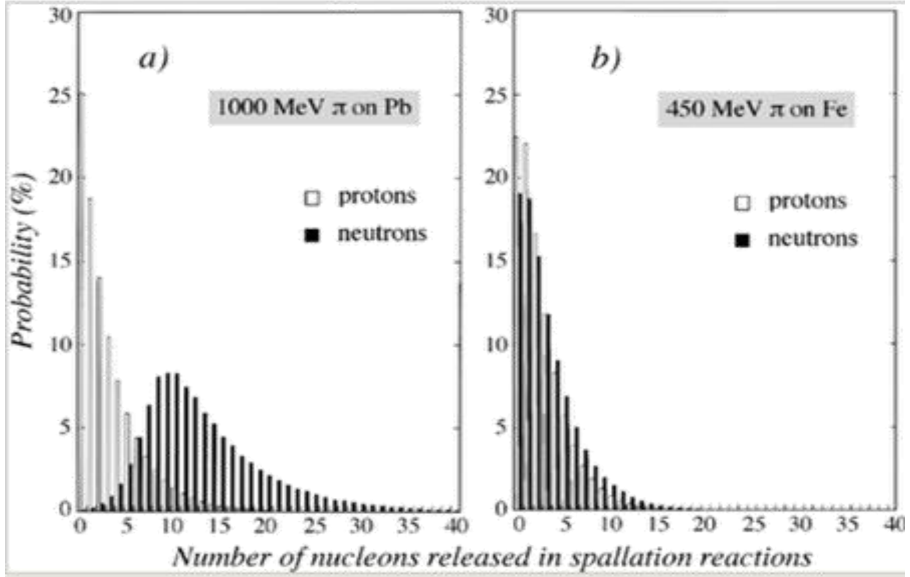


Fig. 2.30. Distribution of the numbers of protons and neutrons produced in spallation reactions induced by 1,000 MeV pions on $^{208}_{82}$Pb (*a*) and by 450 MeV pions on $^{56}_{26}$Fe (*b*).

figure from (2)

3.6.6 Spallation nucleons: p/n distribution

- <200 MeV low probability of proton emission
- protons mainly produced in the fast cascade step
  - Coulomb barrier for protons in nucleus 12 MeV
  - fragments released with a kinetic energy that is a fragment of the binding energy per nucleus 7.9 MeV in lead
  - not many charged particles can escape
  - ratio p/n reflects numerical presence in target nuclei
- *cascade*: 2.7 p + (2.7*126/82=) 4.2 n
  - cascade nucleons induce further spallation reactions
- *evaporation*: 16-7=9 n (all emitted nucleons in evaporation step are neutrons)

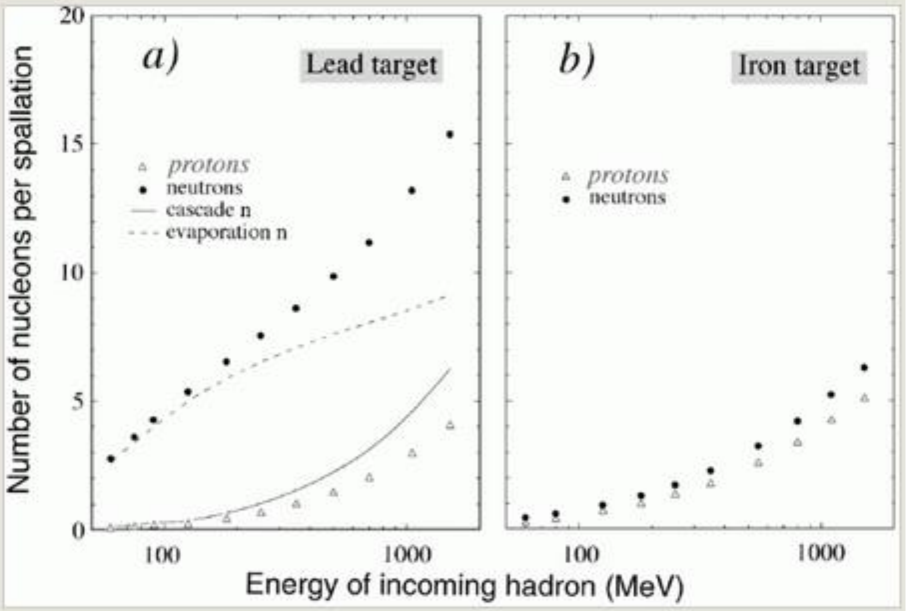


Fig. 2.31. The average numbers of protons and neutrons produced in spallation reactions on $^{208}_{82}$Pb (*a*) or $^{56}_{26}$Fe (*b*), as a function of the energy of the incoming hadron. The neutrons are split up in an evaporation and a cascade component.

figure from (2)

3.6.6 Spallation nucleons: energy distribution

- 16*7.9 = 126 MeV nuclear binding energy
- 9 evaporation neutrons carry ~27 MeV kinetic energy (Maxwell distribution $\frac{dN}{dE} = \sqrt{E}\exp\left(-E/T\right)$ T=2MeV)
- remaining 1000-153=847 MeV shared among the target nucleus and cascade nucleons.
- recoil energy 30 MeV, (847-30)/7=117 MeV/nucleon. Range of 117 MeV protons in Pb: ~2cm. nuclear λ_{int} = 17cm.
  - $\rightarrow$ cascade protons lose their energy by ionization
- remaining (4.2) cascade neutrons will induce new spallation reactions

3.6.7 Spallation nucleons: energy deposition

- all energy is deposited as nuclear binding energy, target recoil energy, ionization by cascade protons or kinetic energy of evaporation neutrons:

Table 2.4 Destination of the 1.3 GeV total energy carried by an average pion produced in hadronic shower development in lead. Energies are in MeV.

	Binding energy	Evaporation n (# neutrons)	Cascade n (# neutrons)	Ionization (# cascade p)	Target recoil
Before first reaction				(300) (π_{in})	
First reaction	126	27 (9)	490 (4.2)	328 (2.8)	30
Generation 2	185	62 (21)	142 (1.6)	98 (1.1)	3
Generation 3	59	21 (7)	36 (0.8)	25 (0.5)	1
Generation 4	24	12 (3)			
Total	394	122 (40)		451 (4.4)	34

table from (2)

3.7 Neutron absorption

Evaporation neutrons lose their (kinetic) energy (~3MeV) by:

- Elastic neutron scattering
  - energies between a few eV and 1 MeV, $\sigma \sim O(b)$: $\rightarrow \lambda \sim O(cm)$
  - energy loss varies between 0 and $f = 4A/(A+1)^2$ (average 50% H, 3-4% Fe, 0.96% for Pb)
  - calorimeters with H and n in this energy range need to take this into account
- Production of α particles
  - between 3 and 20 MeV
- Inelastic neutron scattering
  - neutron brings nucleus into an exited state which then releases γs
  - depends on nuclear level structure
  - insignificant below 1 MeV (Z dependent)
- Neutron capture
  - when neutron lost almost all energy
  - binding energy (invisible energy) is gained back → γ-rays or α particles
  - neutron transforms absorber nucleus into a new type of nucleus

3.8 Quiz

By now you should know:

- the difference between electromagnetic and hadronic showers
- why it takes more material to absorb hadronic showers than electromagnetic showers
- the characteristics of longitudinal and transverse hadronic shower development
- a formula for the electromagnetic fraction of hadronic showers and how to use it to calculate particle multiplicities
- the two phases of nuclear spallation
- how to use Rudstam's formula to estimate p/n distributions in spallation
- some ways in which neutrons lose their energy

If you are unsure, go back to the slides.

Summary

- Shower particles travel in random directions with respect to the particle that initiated the shower
  - orientation of active layers of a sampling calorimeter is not important
- For both electromagnetic and hadronic showers very soft particles (sub-MeV range) carry a major fraction of shower energy
- Em showers:
  - 40% deposited by $e^{+/-}$ with E < 1MeV
  - 1/4 deposited by e^{+}, 3/4 by e^{-} Compton, photoelectrons
  - typical particles: 1 MeV electron, range < 1mm
- Hadron showers:
  - typical particles: 50-100 MeV proton, 3 MeV evaporation neutron
  - range of 100 MeV proton is 1-2cm; neutrons several cm
- Electromagnetic and hadronic showers have different energy deposit mechanisms
  - invisible energy leads to a worse energy resolution for hadronic shower detection
- Differences in shower profiles may be exploited to identify the particles that caused the signals

Please send any questions to eric.van.herwijnen@cern.ch
Problems:

1. Particle interactions with matter that generate showers
   1. The energy stored in one 7 TeV LHC beam is 362 MJ. How much energy will be released in the SHiP target per spill? Assume:
      - Beam intensity: $4*10^{13}$ protons per spill
      - Beam energy: 400 GeV
   2. What is the probability that a multi GeV photon interacts in 1 cm of lead? The same for iron? How much energy will an electron lose? Calculate using the figure on slide 17, compare your answer with slide 19.
   3. How do 10 MeV photons interact in aluminium?
   4. Calculate the effective radiation length and Molière radius of the SHiP target, which consists of 54.1 cm of Mo, 91.5 cm of W, 5.4 cm of Ta, 8.5 cm of H_2O.
2. Electromagnetic showers
   1. Determine the number of X_0 required to contain 95% of 10 GeV electron showers in lead, copper and aluminium. Compare this with 99% containment.
   2. How much extra X_0 are required to contain showers of 20 GeV photons?
   3. How much lead is required to absorb 90% of the energy contained by 100 GeV electrons?
   4. What is the survival probability of 100 GeV muons after 300m of iron?
   5. Does an electromagnetic shower contain more electrons or positrons? Why?
   6. Compare the Molière radius of copper to that of lead and discuss the impact on shower width in both materials.
3. Hadronic showers
   1. What is the major difference between em and hadronic showers?
   2. Calculate the *punch through* probability for 100 GeV protons on a calorimeter with length of 10 λ_{int}
   3. Now do the same for 100 GeV pions.
   4. The cross section for interactions induced by 1 MeV neutrons in lead amounts to ~ 5b. Calculate the mean free path between subsequent interactions for neutrons and compare this to electrons (1 mm) and protons (10 µm) of the same energy. Note the impact on calorimeter design for detecting neutrons.
   5. On slide 50, calculate the energy per cascade nucleon after another generation of spallation. Use figure 31a to estimate the nb of protons and neutrons for 117 MeV.