New Technologies for New Physics

Part I – Basics of Particle Physics

Lecture I

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2020

Building blocks of matter

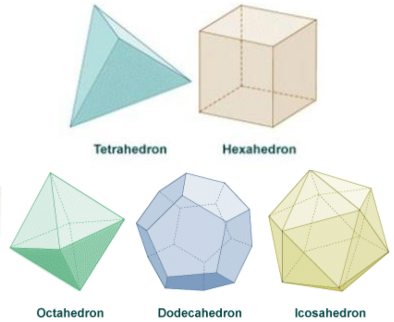
There is an ancient and very fruitful idea in Natural Philosophy – the idea that at different *scales* relevant dynamics can be properly described in terms of different degrees of freedom. This refer to the fact that all but a few objects in Nature can be thought to be *made of* other, in some sense smaller, objects.

For example, *bodies* of all living creatures are made of *cells*, but certainly cell dynamics is governed by different laws than bodies dynamics.

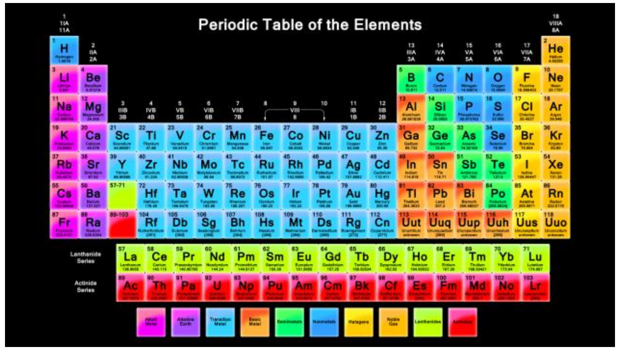
Question (goes back to classical antiquity): what are the ultimate degrees of freedom (i.e. the most fundamental building blocks of matter, those «a few objects» which are not made of other ones)?

Project answers of various epochs (the modern one is on the next slide!):

Four elements

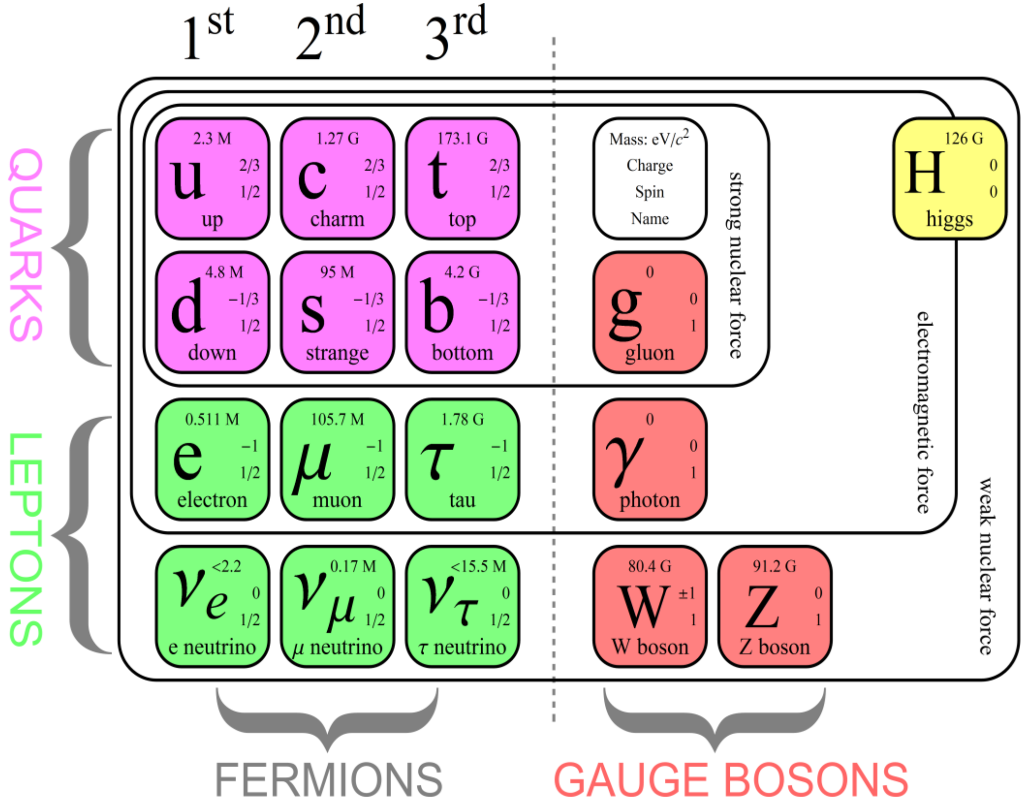


Plato bodies



Mendeleev table

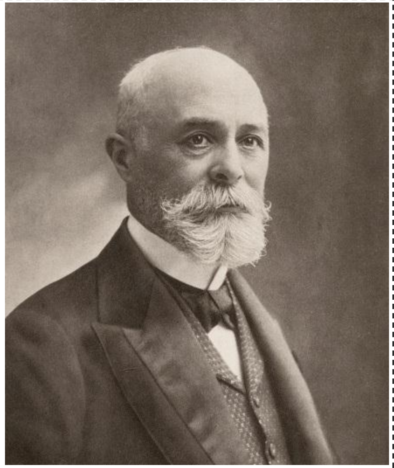
The modern version of the fundamental degrees of freedom zoo looks as follows:



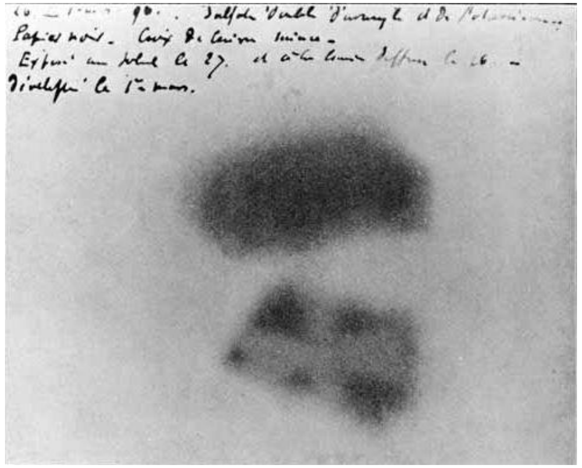
It is commonly known as the Standard Model. It is formulated in terms of elementary particles – objects without any known (at present) internal structure. Each of these particles has a unique name and is characterized by a rather small set of numbers (mass, spin, charges with respect to interactions), reflecting its properties. We will comment on their meaning later, while now remind the main historical steps which led to this picture.

Discovery of subatomic particles – radioactivity

In **1896** **A.H.Becquerel** observed that uranium salts he was using for his studies of phosphorescence emit penetrating radiation without being excited by external source of light.



A.H.Becquerel
(1852-1908)



This is the image of famous photographic plate fogged by exposure to radiation from a uranium salt. One can see the shadow of a metal Maltese Cross placed between the plate and the uranium salt.

The studies of *radioactivity* were continued by **Becquerel's** student **Marie Skłodowska-Curie** (she coined the term) with her husband **Pierre Curie.** They discovered two new radioactive elements – *polonium* and *radium*.

Pierre Curie also showed (using the magnetic field) that some of the radioactive emissions fractions were positively charged, some were negative and some were neutral. They became known as *alpha, beta* and *gamma* radiation.

In **1903**, they all received the Nobel Prize in Physics for discovery and studies of radioactivity. **Marie Skłodowska-Curie** was the first woman to win a Nobel Prize and she is the only person who won two Nobel Prizes in two scientific fields.

Pierre Curie (1859-1906) and
Marie Skłodowska-Curie (1867-1934)

Images from https://en.wikipedia.org/

Discovery of subatomic particles – electron

In **1897** another breakthrough discovery came from J.J.Thomson – the first subatomic particle – *the electron* (the name was introduced by G.J. Stoney earlier as for «single definite quantity of electricity» and came from the Greek word for amber – *ἤλεκτρον*).

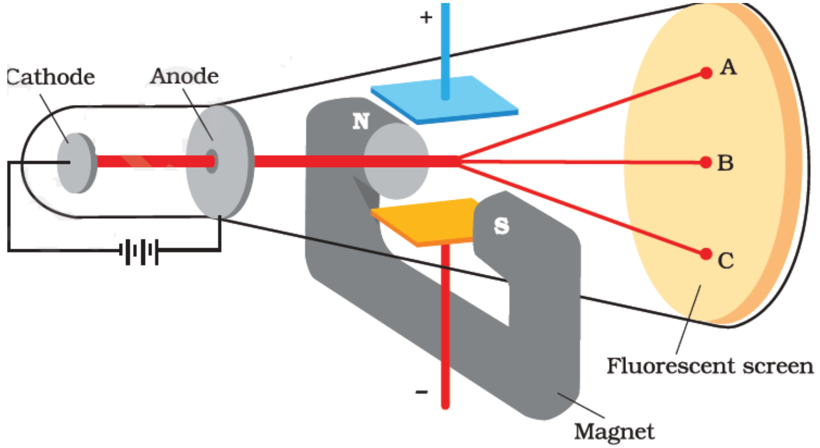


Image from https://hemantmore.org.in/

The main idea was to check whether cathode rays are deflected in electric and magnetic field or not. J.J.Thomson found that they do and comparing the deflections he managed to measure the electron mass-to-charge ratio.

J.J.Thomson
(1856-1940)

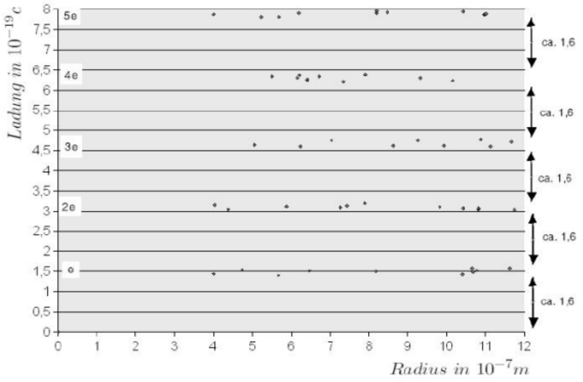
Images from https://en.wikipedia.org/

It was much lower than that of a hydrogen ion, suggesting either that the cathode rays corpuscles were very light and/or very highly charged. Also it was noticed that the rays from every cathode yielded the same mass-to-charge ratio, while for anode rays (flow of positive ions emitted by the anode) mass-to-charge ratios varied from anode to anode.

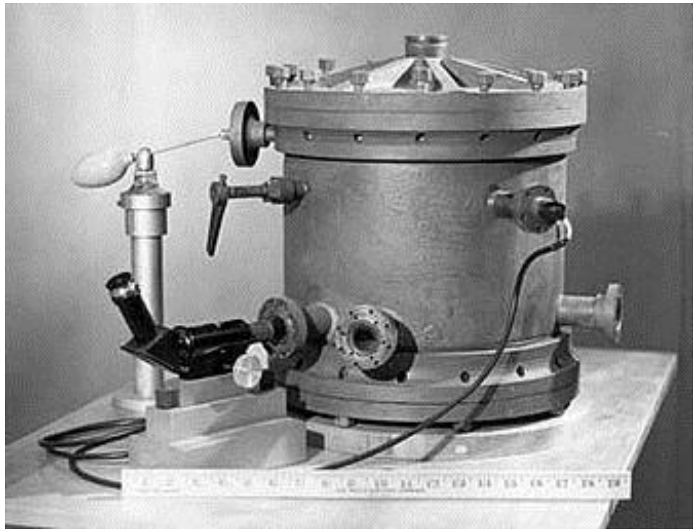
J.J.Thomson got the Nobel Prize in **1906**, it is remarkable that there were also 9 Nobel Prize winners among his students and research assistants (E. Rutherford, Ch.G. Barkla, N. Bohr, M. Born, W.H. Bragg, , F.W. Aston, O.W. Richardson, Ch. Wilson), including his son G.P. Thomson.

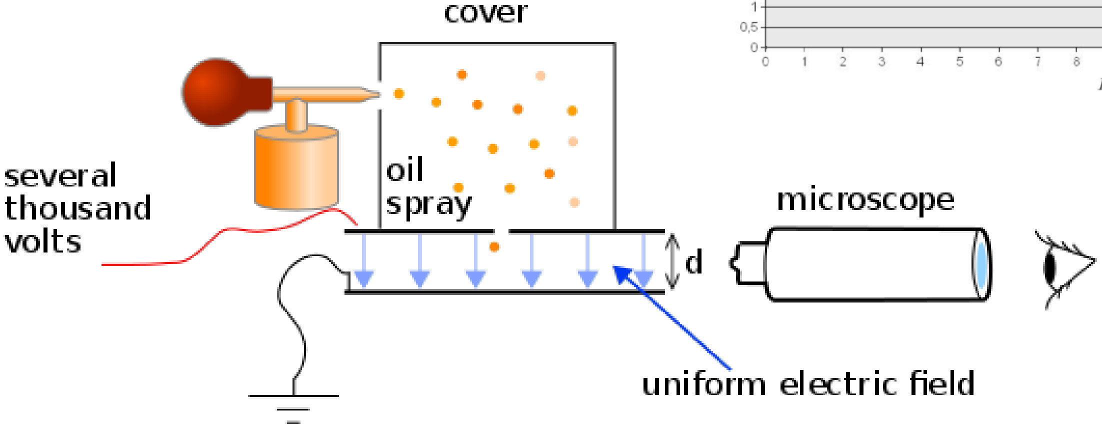
Discovery of subatomic particles – electron charge

In **1909** R.A.Millikan performed the famous «oil drop experiment», aimed to measure elementary electric charge (Nobel Prize **1923**).



R.A.Millikan (1868-1953)





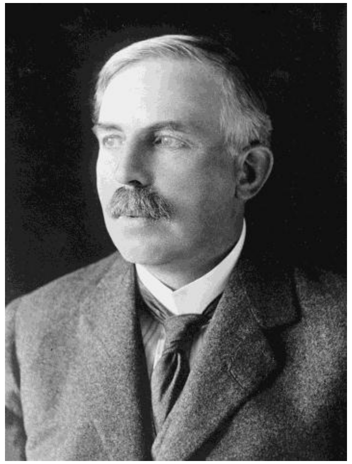
The guiding idea was to balance the gravitational force acting (downwards) on small electrically charged drops of oil by the force directed upwards, caused by applied external electric field. This balance was reached for many droplets of different sizes and charges, repeating the experiment many times. The main fact was that while masses and radii of the droplets were distributed continuously, their charges were integer multiples (rather small) of a certain base value, which was correctly proposed to be the negative charge of a single electron. It was found to be **$1.5924(17)\times10^{-19}$ C**, which is less that 1% difference from the currently accepted value of **$1.602176634\times10^{-19}$ C.**

Since then quantization of electric charge remains one of the great mysteries of fundamental physics.

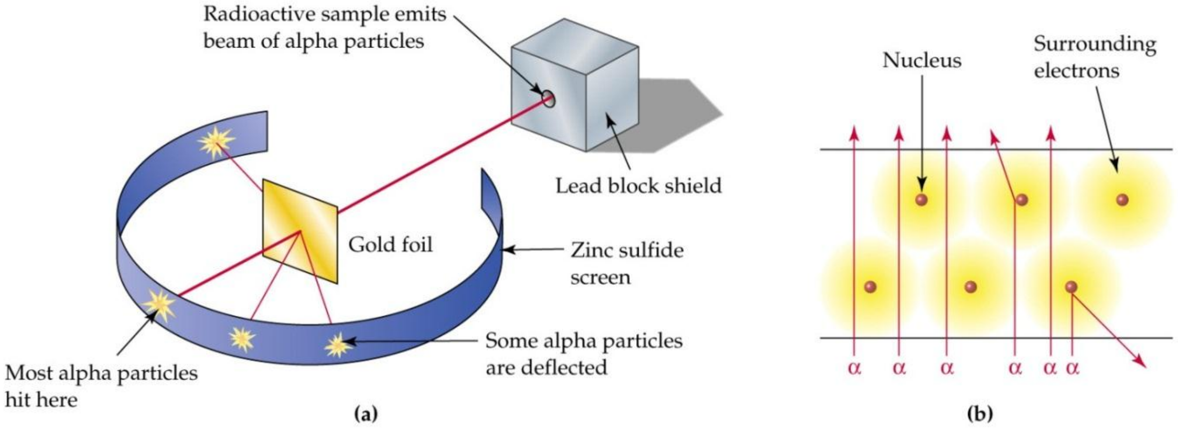
Images from https://en.wikipedia.org/

Discovery of subatomic particles – nucleus

The most famous work of **E.Rutherford** – «gold foil experiment» – was performed by him together with **H.Geiger** and **E.Marsden** in **1909**, after he got the Nobel Prize in chemistry in **1908**, «for his investigations into the disintegration of the elements, and the chemistry of radioactive substances».



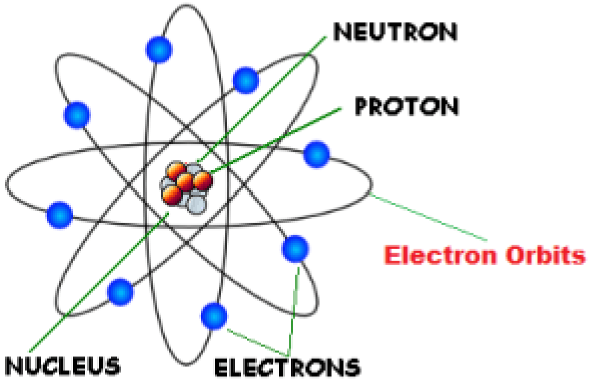
E.Rutherford
(1871-1937)



It was found, contrary to expectations, that some fraction of alpha-particles (bound state of two protons and two neutrons, or He^2 nucleus, in modern language), emitted in some radioactive decays, are deflected by large angles being scattered on thin golden foil.

Rutherford correctly suggested that this is caused by alpha-particle scattering over heavy and compact *atomic nucleus* and this is how the modern view of atom was born.

Among other **Rutherford's** discoveries are the *exponential law* of radioactive decay and radioactive gas *radon*. In **1920** he also postulated that the hydrogen nucleus is a new particle, which he dubbed *the proton*, and suggested the existence of its neutral partner, *the neutron*.



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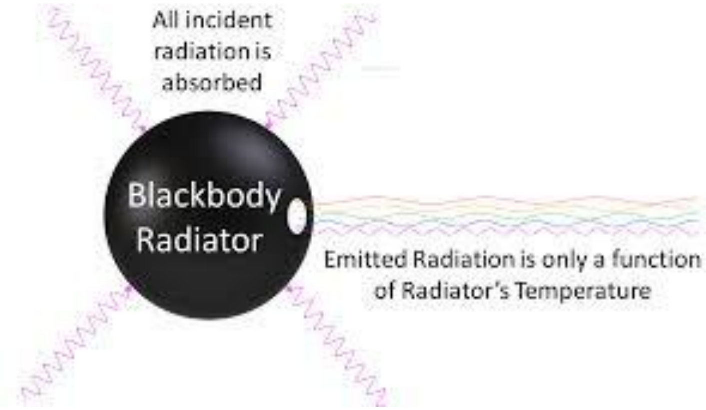
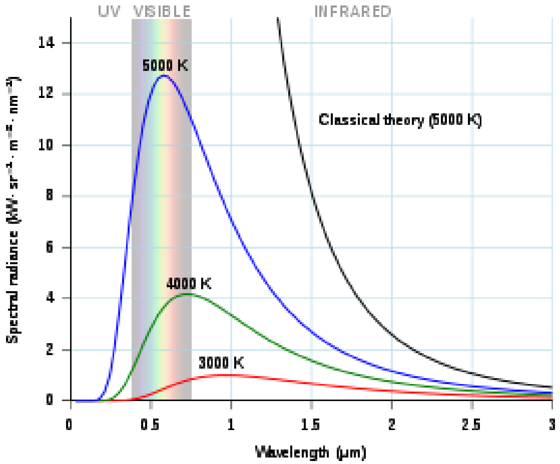
Images from https://en.wikipedia.org/

Birth of quantum theory – Planck law

In **1859** **G.Kirchhoff** formulated the following problem: «how does the intensity of the electromagnetic radiation emitted by a perfect absorber (known as a black body) depend on the frequency of the radiation and the temperature of the body?»

Image from https://shamjijethva97.blogspot.com/,



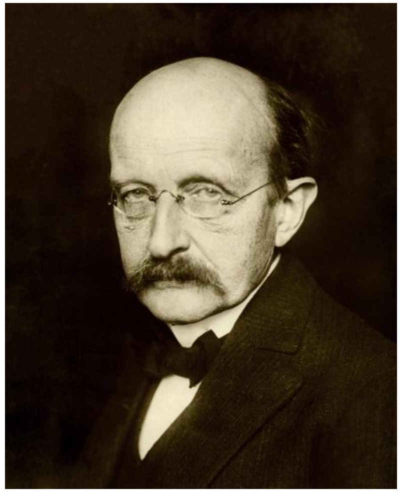


$$\overline{E} = \frac{\int_0^\infty E e^{-E/kT} dE}{\int_0^\infty e^{-E/kT} dE}$$

«Ultraviolet catastrophe»

⇓

$$E = n\varepsilon = nh\nu \Longrightarrow \overline{E} = \frac{\sum_{n=0}^{\infty} n\varepsilon e^{-n\varepsilon/kT}}{\sum_{n=0}^{\infty} e^{-n\varepsilon/kT}}$$



M.Planck
(1858-1947)

Images from https://en.wikipedia.org/

In **1900** **M.Planck** solved the problem introducing the crucial idea of *energy quantum.* According to **Planck's** theory, the spectral radiance of a body is given by:

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$

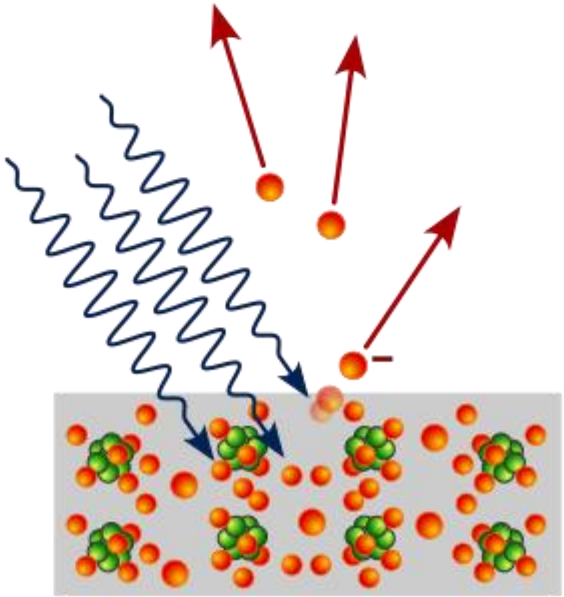
where $h = 6.62 \times 10^{-34}$ **J·s** is a new constant of Nature – Planck's constant. It is often convenient to use $\hbar = h/2\pi$

Young ***Planck*** *was advised by Munich university physics professor* ***P. von Jolly*** *against going into physics, because «In this field, almost everything is already discovered, and all that remains is to fill a few holes»…*

M.Planck got the Nobel Prize for this work in **1918**

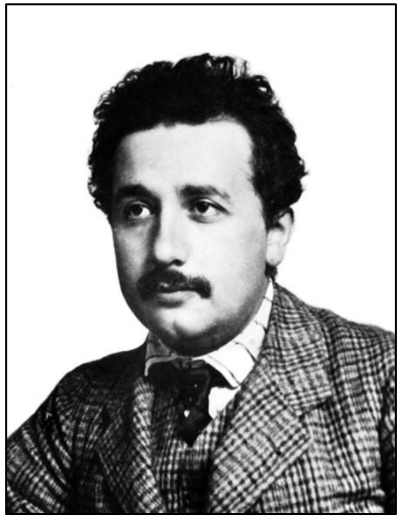
Birth of quantum theory – photoelectric effect

The photoelectric effect is the emission of electrons when electromagnetic radiation hits a material.



Naively one would think that the photoelectric effect is caused by the transfer of energy from the incident radiation (for example, visible light) to an electron, so more intense the light is, larger the kinetic energy of the emitted electrons are.

However experiment demonstrated that it is not the case and electrons' energy depends on light frequency, not intensity. Moreover, there is a threshold frequency (different for different metals) and no electrons are emitted if the incident light has lower frequency, regardless its intensity.



A.Einstein (1879-1954)

In **1905** A.Einstein suggested explanation of photoelectric effect based on the idea of *light quanta* (which we call *photons* now) whose energy is proportional to the frequency: $E = h\nu$

Then the main equation of Einstein's theory has the form:

$$\text{stopping voltage} \longrightarrow qV_o = E_{\max} = h(\nu - \nu_o) \longleftarrow \text{threshold frequency}$$

A.Einstein received the Nobel Prize in **1921** for this work (and not for relativity theory he is most famous for).

Images from https://en.wikipedia.org/

Birth of quantum theory – atom model

The model of atom suggested by E.Rutherford was absolutely incompatible with classical electrodynamics since an electron orbiting the nucleus, should emit electromagnetic waves, lose its energy and finally collapse. The crucial refinements were made by N.Bohr in **1913.**
He postulated:

N.Bohr
(1885-1962)

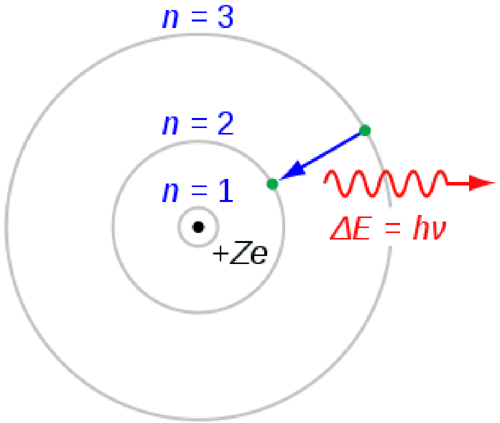
- each electron occupies a certain orbit around the nucleus and the electron cannot have any other orbit in between these discrete ones.
- the sizes of the orbits correspond to electron’s angular momentum as an integral multiple of the Planck's constant. Each orbit has definite electron energy on it and there is the lowest stationary orbit, closest to the nucleus (the ground state).
- electrons can gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting quanta of electromagnetic radiation with a frequency, determined by the energy difference of the orbits.

The theory successfully explained empirical law for the visible spectral lines of a hydrogen atom discovered in **1885** by J.Balmer:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

since in Bohr’s theory the electron energy levels in hydrogen are:

$$E_n = -\frac{2\pi^2 m e^4}{n^2 h^2}$$



Despite many further developments, the basic points of the Bohr model remain valid till now.

Images from https://en.wikipedia.org/

Birth of quantum theory – quantum mechanics

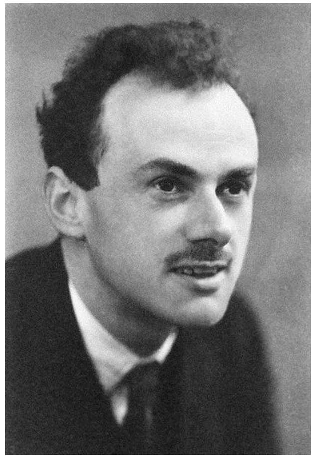
Thus, by **1920s**, there had been collected a set of experimental facts and theoretical ideas, which required departure from the classical physics picture, at least in the domain of small scales. The theory explaining all these facts was built by a few bright people around **1925** and is now known as quantum mechanics. Besides M.Planck, N.Bohr, A.Einstein and some other prominent physicists, the crucial role it this theory construction was played by L.De Broglie, W.Heisenberg, E.Schrödinger, M.Born and P.A.M.Dirac.


L.de Broglie
(1892-1987)


W.Heisenberg
(1901-1976)


E.Schrödinger
(1887-1961)


M.Born
(1882-1970)


P.A.M.Dirac
(1902-1984)

Images from https://en.wikipedia.org/

Systematic presentation of quantum mechanics is beyond our course (but students are welcome to work on the subject by themselves). We present below only the main ideas. Surprisingly, an integral part of relativistic particle physics can be understood without going into quantum mechanical subtleties (see below).

Quiz

1. What are the main experimental milestones of particle physics?

2. How the electron charge was first measured?

3. What are the main points of N.Bohr’s atomic theory?

4. What are the main points of A.Einstein’s photoelectric effect theory?

*

* How does discreteness of energy spectrum in black body emission problem cure ultraviolet catastrophe?

* Calculate the long-wavelength threshold of photoeffect in copper (photoelectric work function for copper is **4.5 eV**).

* In Bohr’s atomic theory, estimate the relative electron energy levels shifts due to finiteness of the proton mass.

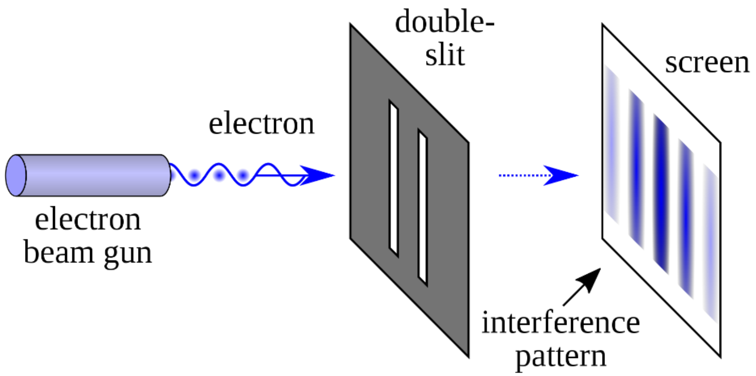
Basics of quantum mechanics – particle-wave duality

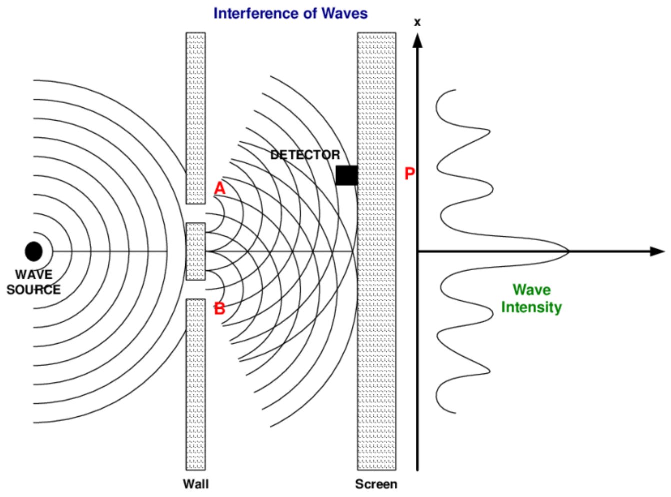
In **1923** L.de Broglie proposed that Planck-Einstein relations (introduced for light quanta, photons) $E = h\nu = \hbar\omega \qquad p = \hbar k \qquad \hbar = h/2\pi$

should be extended to all particles (and even material bodies). He proposed that a material particle moving with momentum p has a wavelength

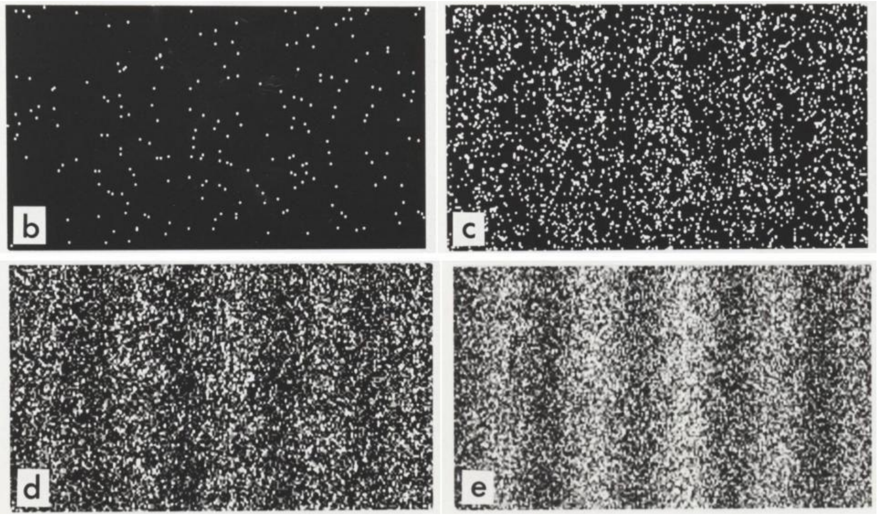
$$\lambda = \frac{h}{p}$$

This is not a usual wave known in classical physics but a kind of quantum probability wave. But they can interfere like normal classical waves.





It is important that this is not interference of different electrons with each other, but self-interference of de Broglie wave of each electron with itself.



Images from https://en.wikipedia.org/

Basics of quantum mechanics - uncertainty

Another important physical law was formulated in **1925** by W. Heisenberg and is known as uncertainty principle. It poses a fundamental limit to the precision with which the values for certain pairs of physical quantities can be predicted or measured. Such pairs are known as complementary or canonically conjugate variables.

The pair most important in practice is the coordinate and the momentum. If a particle is localized in space within the region with the size Δx , its momentum cannot be defined better than with the precision Δp and

$$\Delta x \Delta p \geq \hbar$$

In a sense, the uncertainty principle is inherent in the properties of all wave-like systems, and its physical cause in quantum mechanics is nothing but de Broglie wave nature of all quantum matter. However, it is worth stressing that the uncertainty principle is a fundamental law of Nature and not an instrumental statement about measurement process, which can be overcome by further development of observational technologies.

Uncertainty principle had put a firm basis on statistical physics, since it states that phase space of any system has an inherent discreet structure, making possible the correct counting of states.

In modern particle physics detectors, uncertainties in detected relativistic particle positions are of order of **10^{-6} m** and worse (i.e. larger), while uncertainties in momentum are at best of order **1 кeV** and worse, which keeps their product at least a factor **10^4** above the quantum uncertainty limit. However in future devices it can become relevant, as it already is in atomic physics and quantum optics.

Basics of quantum theory - transitions

The key notion for any quantum systems in a state. In quantum theory, a quantum state is a mathematical object, corresponding to a given physical system prepared in a particular way. In many cases it is convenient to represent it as a vector in some Hilbert space (space of states). It provides a probability distribution for the results of any measurement process one can do with the system.

The outcome of a single measurement is considered in quantum mechanics as principally unpredictable (besides special cases when probability of a given outcome happens to be unity), contrary to the classical physics. Among the main processes of interest in quantum mechanics, relevant in relativistic case (which is of central interest for particle physics) is the transition between the initial and the final states.

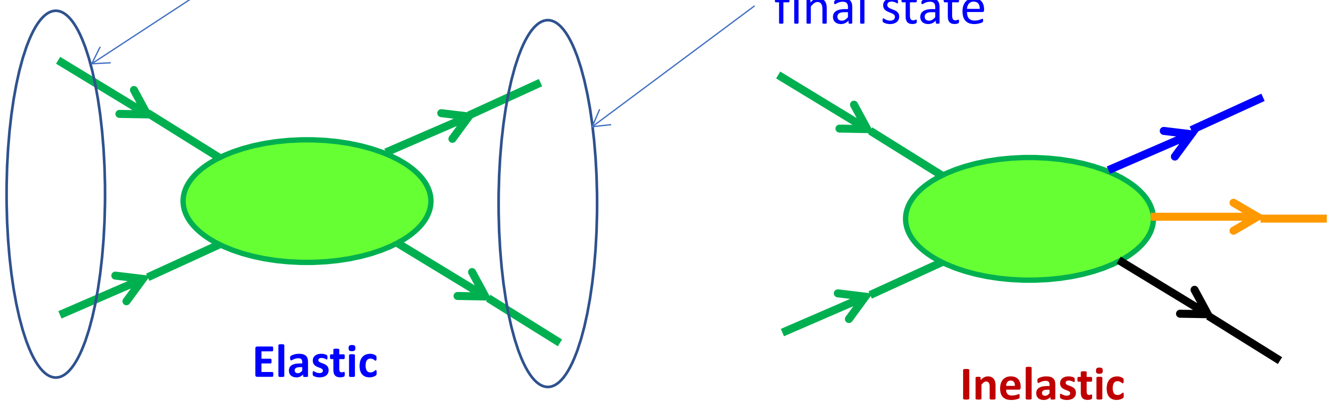
Example: scattering

initial state

$$|i\rangle \to |f\rangle = \textcircled{S}|i\rangle$$

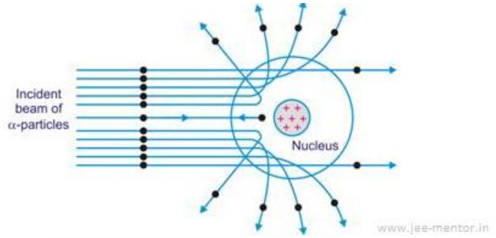
final state

S-matrix – operator, encoding dynamics of the process and transforming initial states to final states

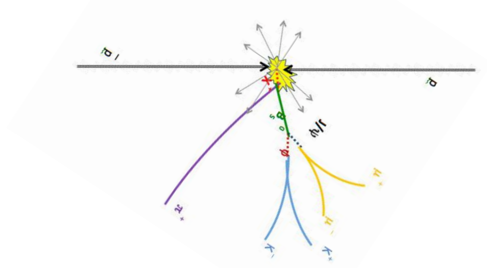


Elastic

Inelastic



Fixed target collision mode



Collider collision mode

Basics of quantum theory - amplitudes

Matrix elements of the S-matrix in the vector basis of states are known as transition amplitudes.

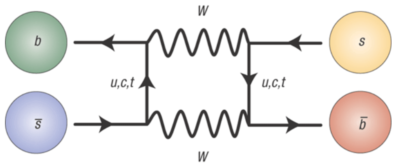
$$A_{if} = \langle f | S | i \rangle$$

These are complex numbers (i.e. they have real and imaginary parts). It is assumed that each state is characterized by a set of numbers (for example, momenta of two colliding particles of a given type), so the amplitudes are some functions of these numbers. There is sophisticated theoretical machinery how to compute these functions, in many cases they are with good accuracy represented as integrals in momentum space:

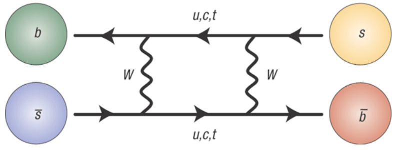
$$A(p_{in}, p_{out}) = \int dq\, V_1[q,p] \cdot D_2[q,p] \cdot \ldots$$

The experimentally observable, however, are not amplitudes but transition probabilities: $P \propto |A|^2$

It is highly nontrivial that there can be more than one different amplitudes for the same initial and final states.



$$= A_1$$



$$= A_2$$

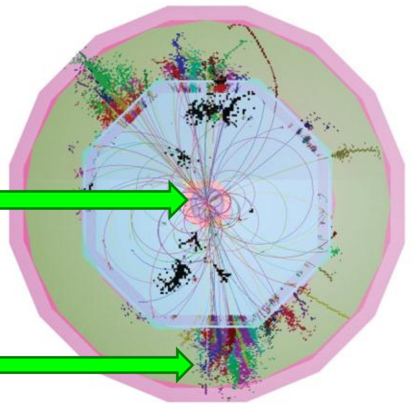
$$P = |A_1 + A_2|^2$$

Interference

to be complex is important!

In real experiments these probabilities have to be corrected to the detector performance.

Result = theory × detector

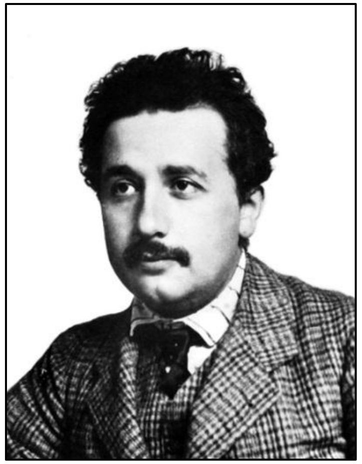


Relativity – roots

There is the second pillar, besides quantum theory, of modern particle physics. It is special relativity.
Deeply rooted in the development of physics in the **XIXth** century, its main ingredients were discovered by H.Lorentz, A.Poincare and other scientists and finally formulated in modern form by A.Einstein in **1905**.

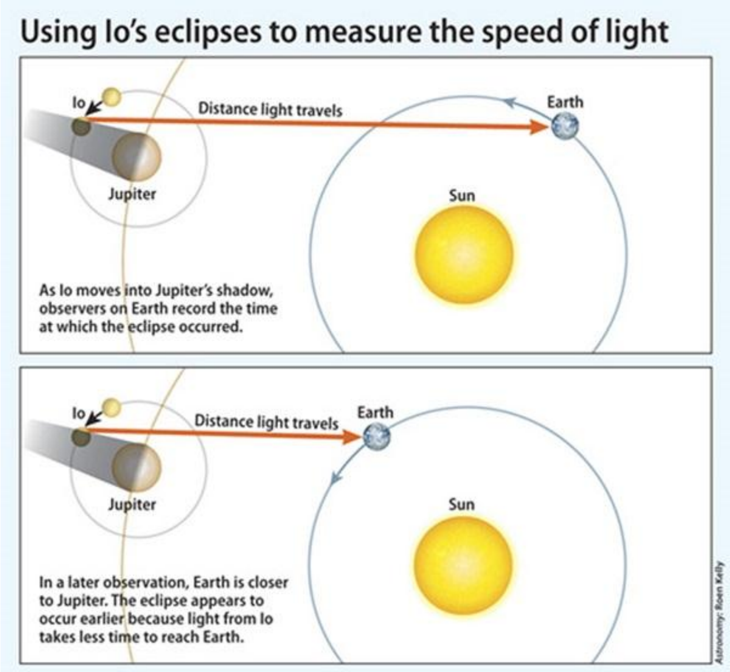
H.Lorentz
(1853-1928)

H. Poincaré
(1854-1912)



A.Einstein
(1879-1954)

On of the first steps towards special relativity was done in **1676** by O. Rømer, who found, by observation of Jupiter's satellite Io eclipses that the light takes about **22 minutes** to travel the distance equal to the Earth orbit diameter. This is about **25%** off the exact value – great result for the science of the **XVIIth** century! Next notable measurements in the **XVIIIth** and the **XIXth** centures included works of J. Bradley, H. Fizeau, L. Foucault and other scientists.

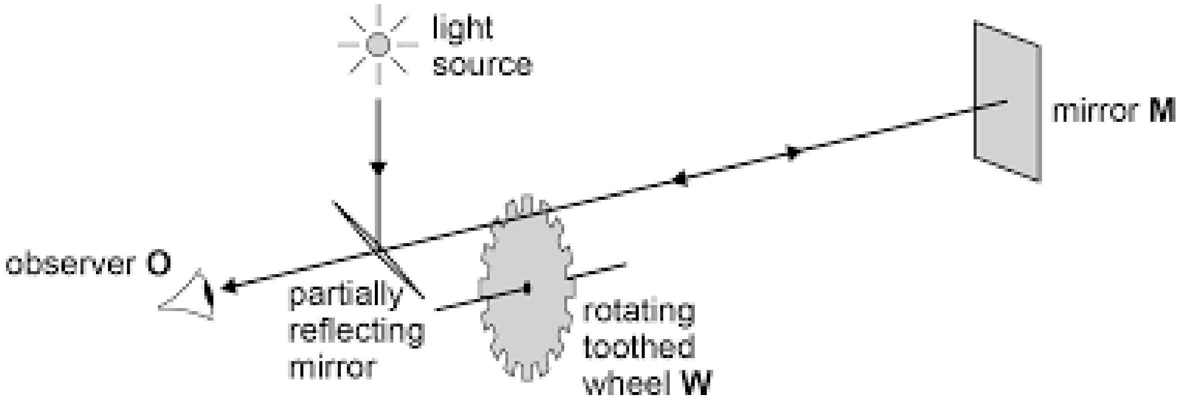


Using Io's eclipses to measure the speed of light

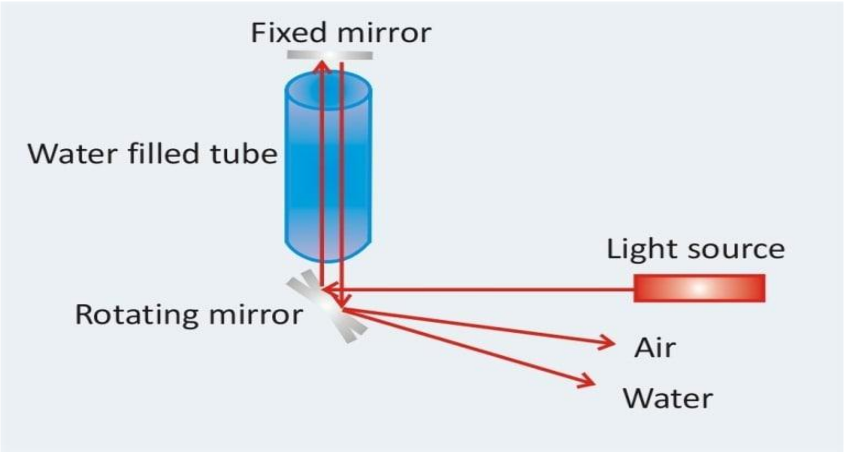
Images from https://en.wikipedia.org/

The currently accepted system of units takes speed of light to be equal **299 792 458 meters per second** exactly. It just means that one meter is defined as the length of the path travelled by light in **1/ 299 792 458** of a second. Together with the Planck constant **ħ**, this is another fundamental constant of our world, usually denoted as **c**.

ħ **c**



H. Fizeau's measurement of the speed of light (**1848-1849**) by the «rotating toothed wheel» experiment



L. Foucault's measurement of the speed of light (**1862**) by the «rotating mirror» experiment

1675	O. Rømer and C. Huygens, moons of Jupiter	220000	–27% error
1729	J. Bradley, aberration of light	301000	+0.40% error
1849	H. Fizeau, rotating toothed wheel	315000	+5.1% error
1862	L.Foucault, rotating mirror	298000±500	–0.60% error
1907	E.Rosa and N.Dorsey, EM constants	299710±30	–280 ppm error
1926	A. Michelson, rotating mirror	299796±4	+12 ppm error
1950	L.Essen and A.Gordon-Smith, cavity resonator	299792.5±3.0	+0.14 ppm error
1972	K.Evenson *et al.*, laser interferometry	299792.4562±0.0011	–0.006 ppm error
1983	17th CGPM, definition of the metre	299792.458 (exact)	exact, by definition

Images from https://en.wikipedia.org/

Relativity – electrodynamics

J.C. Maxwell
(1831-1879)

Another root of relativity is the theory of electric and magnetic phenomena. Their studies were among the «hottest topics» in physics in the second half of the **XVIIIth** – the first half of the **XIXth** centures. B.Franklin, L.Galvani, A.Volta, C.A. Coulomb, G.Ohm, H.C. Ørsted, C.Gauss, M.Faradey, A.Amper and other prominent scientists developed this branch of physics.

This line of research was crowned by J.C.Maxwell, who formulated a general theory of electromagnetic phenomena around **1870th**.

Leaving aside processes where quantum dynamics is important, this theory in its essence – «Maxwell equations» – remains the framework of our current understanding of electromagnetism.

Maxwell found that his equations for electromagnetic field had solutions corresponding to transverse waves, having velocity, close to the velocity of light, known at the time.

«The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws».

J.C.Maxwell, «A Dynamical Theory of the Electromagnetic Field», **1864**

«The special theory of relativity owes its origin to Maxwell's equations of the electromagnetic field».

A. Einstein , «Autobiographisches», **1949**

Images from https://en.wikipedia.org/

Relativity – two postulates

In modern approach, special relativity is based on two «postulates» – statements about properties of Nature, those consequences are to be compared with the experiment.

- Relativity principle

There are frames of reference (called inertial) in which isolated passive body keeps moving with constant speed (or stay at rest) unless other bodies interact with it. Frames of references moving with constant speed with respect to inertial frame, are also inertial. No any experiment (inside the system) is able to detect the fact of movement of one inertial frame to another, i.e. laws of Nature are identical in all inertial frames.

This principle is also valid in Galileo-Newton mechanics

- Limiting speed

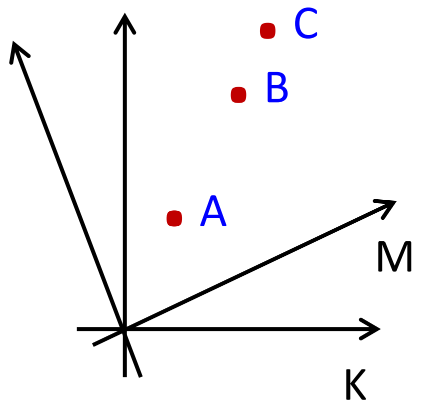
There is a limiting speed **c** for any signal, i.e. interaction, realizing causal connection.

- ✓ No this principle in Galileo-Newton mechanics (i.e. interaction can be instantaneous)
- ✓ According to relativity principle, this limiting speed is one and the same in all inertial frames
- ✓ We will see that it coincides with the speed of light in a vacuum

Important: the speed in question is the «signal» speed, speed of information or material object transmittion, and not wave phase speed, or wave group speed, or «sunlight spot» speed, or alike. Any of the latter can be greater than **c**.

Relativity and invariance

Relativity brings another fruitful idea – the importance of distinction between those parts of a phenomenon description, which varies between different reference frame, and those which are invariant.

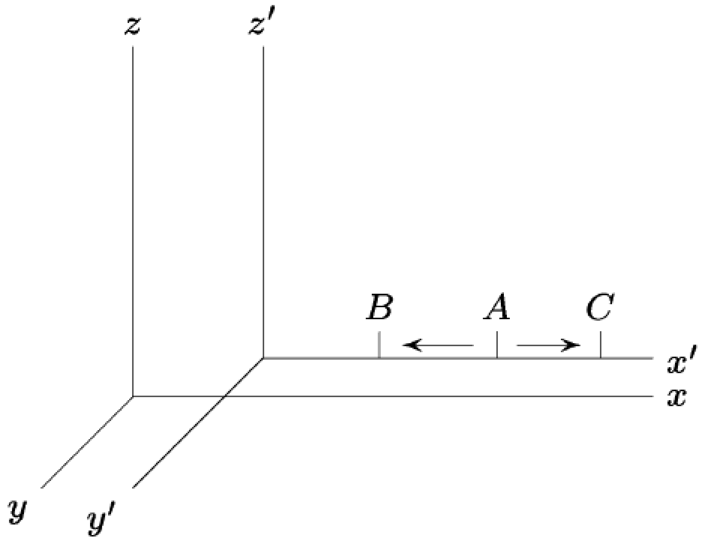


What is different in coordinate systems К and M	What is invariant in К and M
• Coordinates of the points A, B, C • Angles between $\overrightarrow{AB}$ and axes.	• Distances between points • The fact that A, B and C lie on a line.

Let one frame moves along ***x*** axis with the velocity *V*

Then in Galileo-Newton mechanics

$$x = x' + Vt, \quad y = y', \quad z = z', \quad t = t'$$



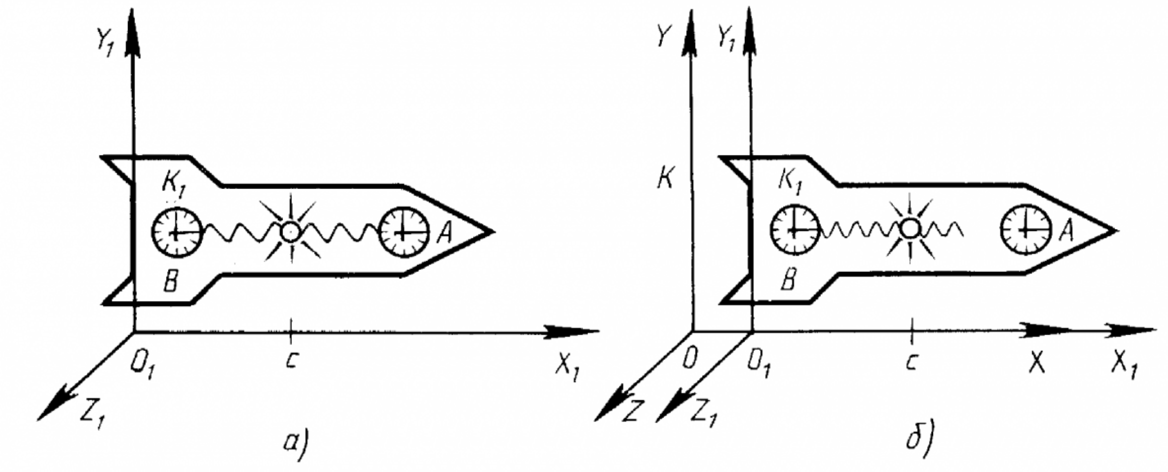
If from A to B and C two bullets are shot simultaneously with the velocities *v* and –*v* (with respect to the point A), both observers, the one moving and another at rest in A, will see that the bullets reach B and C also simultaneously.

Absolute time

Relativity – interval

Relativity postulates are incompatible with absolute time.

Let from C to A and B two signals are sent simultaneously. They travel with the limiting speed **c**, universal for all inertial frames. For observer at rest in the rocket, the signals come to B and C simultaneously. But for the one outside the moving rocket, the signal came to B before it came to A.



Relativity of simultaneity

Mathematically,

the invariance of limiting speed is expressed as $(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2 = 0$

in laboratory frame and $(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2 - c^2(t'_2 - t'_1)^2 = 0$

in a moving (with respect to the laboratory one) frame.

In general, the quantity $s_{12} = [c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2]^{1/2}$

for two events $x_1,\ y_1,\ z_1,\ t_1$ and $x_2,\ y_2,\ z_2,\ t_2$

is known as space-time interval between events. In differential form $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$

It can be proven that interval is invariant, i.e. it is the same in all inertial frames.

Relativity – time dilation

Invariance of the interval has many immediate consequences. For example, for a moving clock one has

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = c^2dt'^2$$

it gives for the proper time (i.e. the time in the reference frame where the clock is at rest) $dt' = \frac{ds}{c} = dt\sqrt{1 - \frac{v^2}{c^2}}$

Moving clock runs slower than one at rest

There is a beautiful example of this effect in Nature – decay of cosmic muons. Muons are charged leptons of the second generation with the mass **105.65 MeV** and lifetime **2.2 microseconds**. Naively they can run about $l_0 = c\tau = 660\ m$

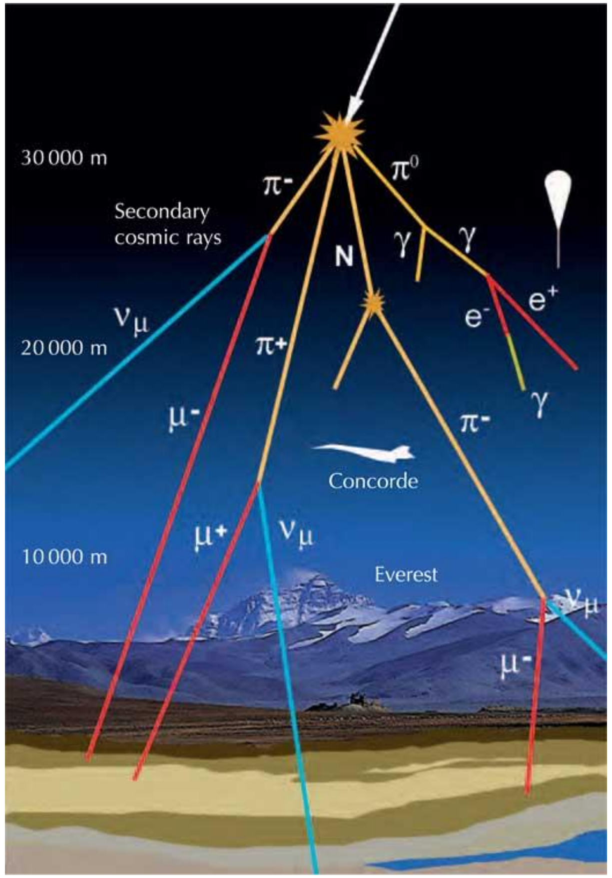
In reality muons are the main component of particles of cosmic origin at the sea level with the flux density about **170 m^{-2} sec^{-1}**

It is convenient to introduce Lorentz-factor $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

For **5 GeV** muon Lorentz-factor **γ = 47** and $l = c\tau\gamma = 31\ km$

For Earth orbiting around the Sun, **γ=1.000000005**

But for protons in Large Hadron Collider at CERN **γ=7463**



Images from https://en.wikipedia.org/

Relativity – Lorentz transformations

Coordinate transformations, which keeps interval invariant, had been found (or guessed) before the special relativity by W.Voigt (in his studies of Doppler effect), J.Larmor (Maxwell equations symmetries), H.Lorentz, A.Poincare. They are known in modern literature as Lorentz transformations.

In two dimensions, invariance of the interval $(ct)^2 - x^2$ (assuming linearity) is reached by hyperbolic rotation

$$x = x'\,\mathrm{ch}\,\psi + ct'\,\mathrm{sh}\,\psi$$

$$ct = x'\,\mathrm{sh}\,\psi + ct'\,\mathrm{ch}\,\psi$$

If one frame moves with the speed ***V*** with respect to another frame

$$\frac{x}{ct} = \mathrm{th}\,\boxed{\psi} \longleftarrow \text{rapidity}$$

$$\mathrm{th}\,\psi = V/c \qquad \mathrm{ch}^2\psi - \mathrm{sh}^2\psi = 1$$

$$\mathrm{sh}\,\psi = \frac{V/c}{\sqrt{1 - V^2/c^2}} \qquad \mathrm{ch}\,\psi = \frac{1}{\sqrt{1 - V^2/c^2}}$$

In 3+1 dimensions

$$x = \frac{x' + Vt'}{\sqrt{1 - V^2/c^2}}, \quad y = y', \quad z = z', \quad t = \frac{t' + (V/c^2)x'}{\sqrt{1 - V^2/c^2}}$$

In nonrelativistic limit ***V << c***

$$x = x' + Vt', \quad y = y', \quad z = z', \quad t = t' + \frac{V}{c^2}x'$$

Relativity – group structure

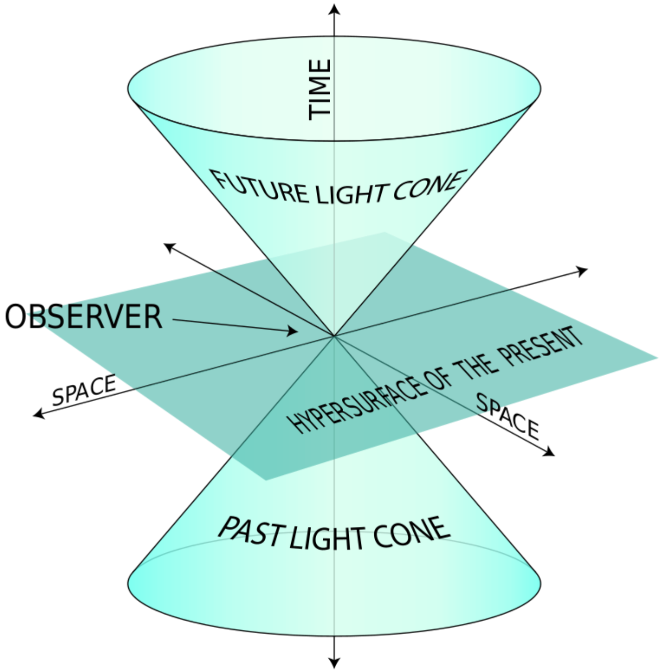
In mathematical language, Lorentz transformation form the group – group of homogeneous linear coordinate transformations of **3+1** dimensional space-time, which leaves interval invariant. This is **6**-parameter group (**3** space rotations + **3** boosts + reflections) denoted as **O(3,1)**.

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} \qquad g_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} = g_{\alpha\beta} \qquad (g^{ik}) = (g_{ik}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Inhomogeneous Lorentz group includes translations and is known as Poincare group

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}$$

Structure of these groups, their representations and corresponding Lie algebras play crucial role in physics of relativistic systems. In particular, two main parameters characterizing every particle – its mass and spin – correspond to two Casimir operators of Poincare group.



Space-time in special relativity is divided into causally connected and disconnected regions – the structure known as light cone. World-line of any material particle lies inside its future light-cone.

Images from https://en.wikipedia.org/

Relativity – dynamics

Dynamics of relativistic objects follows from the Lorentz group structure. Natural generalization of velocity is 4-velocity

$$u^i = \frac{dx^i}{ds} \qquad u^i = \left(\frac{1}{\sqrt{1-v^2/c^2}}, \frac{\mathbf{v}}{c\sqrt{1-v^2/c^2}}\right)$$

In 4-dimensional sense, any particle always moves with the velocity **c**: $u^2 = u_i u^i = 1$

If 3-velocity (along spatial directions) is nonzero, «velocity» along temporal axis is less than **c** – this is time dilation in another language.

Action for relativistic particle: $S = -\alpha \int_a^b ds \quad \Longleftrightarrow \quad S = \int_{t_1}^{t_2} L\, dt$

$$ds = c\, dt \sqrt{1 - v^2/c^2}$$

$$S = -mc \int_a^b ds \qquad L = -mc^2\sqrt{1 - \frac{v^2}{c^2}}$$

Momentum: $\mathbf{p} = \partial L / \partial \mathbf{v}$

Energy: $\mathscr{E} = \mathbf{p}\mathbf{v} - L$

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}} \qquad \boxed{\mathscr{E} \approx mc^2} + \frac{mv^2}{2} \qquad \mathscr{E} = \frac{mc^2}{\sqrt{1-v^2/c^2}} \qquad \mathbf{p} = \frac{\mathscr{E}\mathbf{v}}{c^2}$$

Relativity – mass and energy

Relativistic dispersion law $E^2 = p^2c^2 + m^2c^4$

Ultra-relativistic limit $E = pc$

Non-relativistic limit $E = mc^2 + \dfrac{p^2}{2m}$

In ultra-relativistic limit the energy is proportional to the first power of momentum (and not momentum squared as in non-relativistic case)

4-momentum: $p^i = \left(\frac{\mathscr{E}}{c}, \mathbf{p}\right)$

$$p^i p_i = m^2c^2$$

vector components

scalar

Mass is Lorentz-invariant and does not change with body’s velocity. At the same time, mass and energy transform to each other

Mass to energy

When **1 gram** of matter and **1 gram** of antimatter at rest annihilate, **1.8×10^{14} Joules** (**43 кtons TNT**) of energy is produced.

Energy to mass

When two protons at Large Hadron Collider with energies **7000 GeV** each collides, some part of their energy goes to Higgs boson mass **125 GeV / c^2**

For «human» scale processes effects are negligible, for example when ice melts, ***Δm/m*** **= 3.7×10^{-12}**

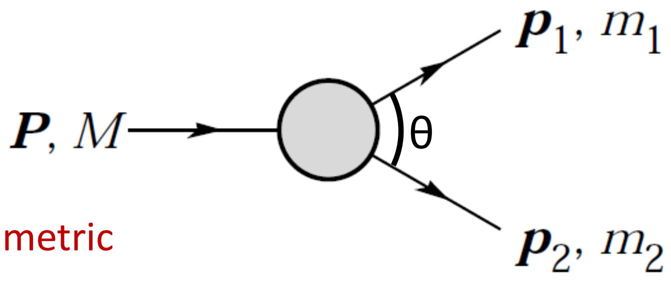
Relativity – kinematics

It is very convenient in relativistic physics to use system of units where **c=1**. Velocity is dimensionless in such system, while energy, mass and momentum have the same dimensionality.

Objects which are experimentally registered in detectors are (relatively) long-lived particles (e, μ, π, p, K, γ, n).

The reconstruction problem appears: having measured types (i.e. masses) and momenta (including both absolute values and angles) of the final particles, reconstruct the mass of the initial one.

The simplest example: decay **1 → 2**



$$P = p_1 + p_2$$

$$P^2 = (p_1 + p_2)^{\textcircled{2}} \longleftarrow \text{in Minkowskii metric}$$

$$M^2 = m_1^2 + m_2^2 + 2\sqrt{(m_1^2 + p_1^2)(m_2^2 + p_2^2)} - 2p_1p_2\cos\theta$$

$$(E, \mathbf{P}) = (E_1, \mathbf{p}_1) + (E_2, \mathbf{p}_2)$$

in the initial particle rest frame:

$$(M, 0) = (E_{10}, \mathbf{p}_{10}) + (E_{20}, \mathbf{p}_{20})$$

$$\begin{cases} M = E_{10} + E_{20} \\ \mathbf{p}_{10} + \mathbf{p}_{20} = 0 \end{cases}$$

Kinematics is fully fixed in **1 → 2** decay.

With more particles in the final state the phase space appears – a set of final particles kinematical configurations allowed by conservation laws.

$$E_{10} = \frac{M^2 + m_1^2 - m_2^2}{2M} \qquad E_{20} = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

Quiz

1. What is the Heisenberg's uncertainty principle?

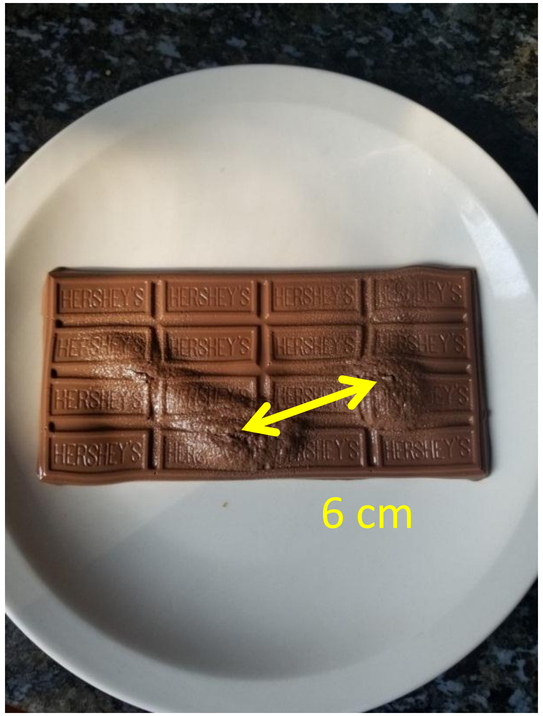
2. What are the special relativity postulates?

3. What are the main differences between relativistic and nonrelativistic kinematics?

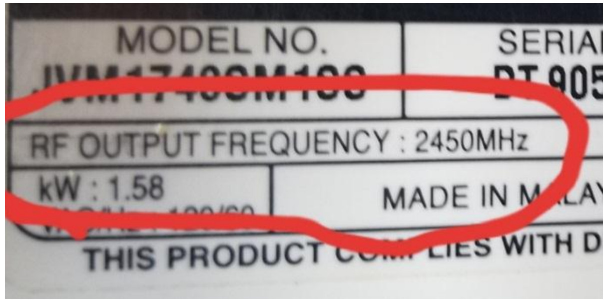
4. How to reconstruct the mass of decayed particle? If not at rest?

*

* Someone placed a chocolate into microwave oven (non-rotating regime) and found the distance between the most heated zones to be about **6 cm**. Estimate the speed of light (the oven frequency is **2450 MHz**).



Images from https://t.me/sijekotech/



Quantum field theory

Quantum theory of relativistic objects was built for a few decades by many people starting from **1928** when P.A.M.Dirac wrote down his famous equation. This quantum field theory (QFT) put quantum particle-wave duality on firm mathematical basis – particles are considered as quanta of the corresponding fields. Nowadays this is a well developed theoretical machinery, intensively using computer algebra methods for computations. One of the key physical quantities here is a Compton wavelength (due to A.Compton, Nobel Prize **1927**). It is, by definition, equal to the wavelength of the photon with the same mass, as the mass of the particle (for example, it is **2.43×10^{-12} m** for electron, which is about **20** times smaller than the Bohr radius):

$$\lambda = \frac{h}{mc}$$

The Compton wavelength expresses a fundamental limitation on measuring the position of a particle, taking into account both quantum mechanics and special relativity. In QFT, a particle cannot be localized in space with the precision better than its Compton wavelength, and an attempt to do it will result in creation of other particles.

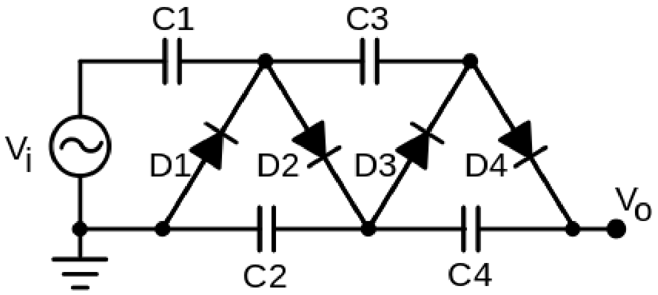
For ultra-relativistic particles their de Broglie wavelength is much smaller than the Compton wavelength. It is the main reason why one can talk about well localized particle trajectories and interaction points in this kinematical domain as if the particles behave themselves indeed like classical small hard balls.

Larger the momentum → the particle is more «particle-like»

Tools - accelerators

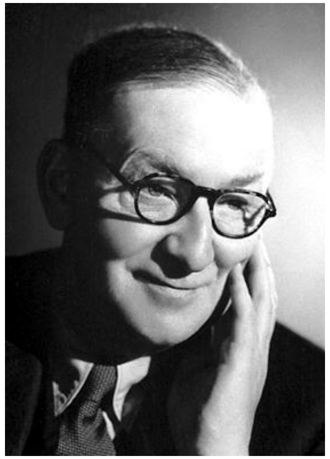
Most important relativistic objects we are dealing with are subatomic particles (and of course photons – electromagnetic field quanta – they are always relativistic). History of these particles discoveries goes in parallel with the development of two main group of tools: accelerators and detectors. The important piece of knowledge was obtained also be studies of naturally accelerated particles in the cosmic rays (see below).

There have been many different types of accelerators of charged particles in the **XXth** century. One of the first was constructed in **1930s** by J.Cockcroft (PhD student of E.Rutherford and, later, assistant of P.Kapitza) and E.Walton. The main motivation was to bombard the nucleus by protons of energies high enough to overcome electrostatic repulsion.



With the protons accelerated up to **1 MeV** they were able to succeed for the first time splitting the atom:

$${}^{7}_{3}\mathrm{Li} + \mathrm{p} \rightarrow 2\,{}^{4}_{2}\mathrm{He} + 17.2\ \mathrm{MeV}$$


J.Cockcroft
(1897-1967)


E.Walton
(1903-1995)

They received the Nobel Prize in **1951** for this work.

Images from https://en.wikipedia.org/

Tools – cyclotron

Next advance came (approximately at the same time) when E.O.Lawrence with his group constructed cyclotron – a device using cycling acceleration in the magnetic field.

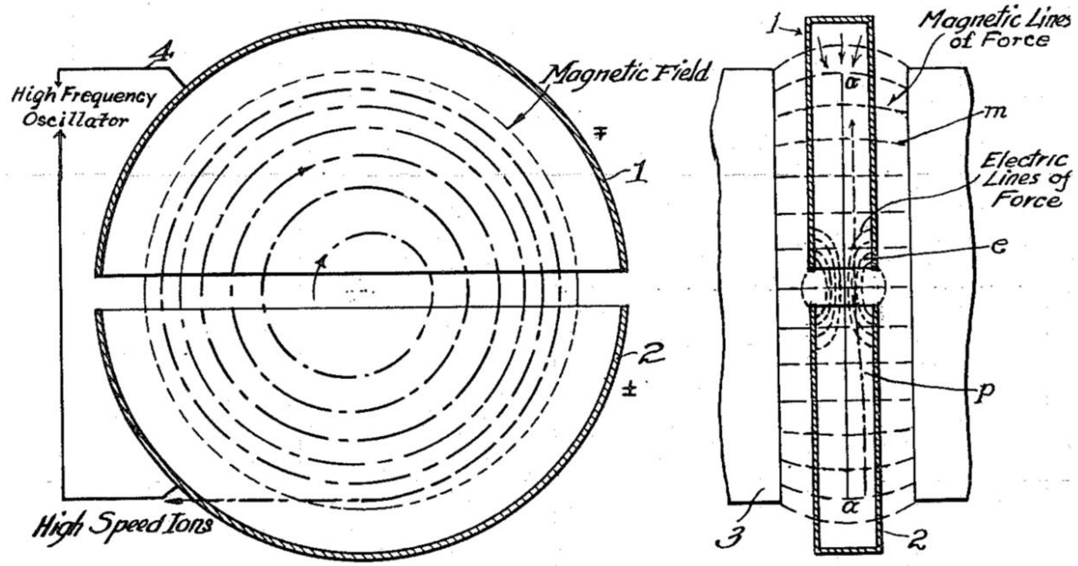


Diagram of cyclotron operation from E.Lawrence's 1934 U.S. Patent 1,948,384

A key idea is to switch the electric field by alternating voltage applied between two hollow D-shaped sheet metal electrodes known as «dees» inside a vacuum chamber. The frequency matches particle's cyclotron resonance frequency so that the particles make one circuit during a single cycle of the voltage. This allows cyclic acceleration.

E.O.Lawrence (1901-1958)

The world's largest cyclotron in Canada's laboratory TRIUMF has **18 meters** diameter and **4,000 tons** magnet producing **0.46 Tesla** field. It accelerates protons up to **500 MeV.**

The first European cyclotron was proposed by G.Gamow and L.Mysovskii in **1932** and constructed in Leningrad in **1937**. Cyclotrons were the most powerful particle accelerators until the **1950s** when they were superseded by synchrotrons.

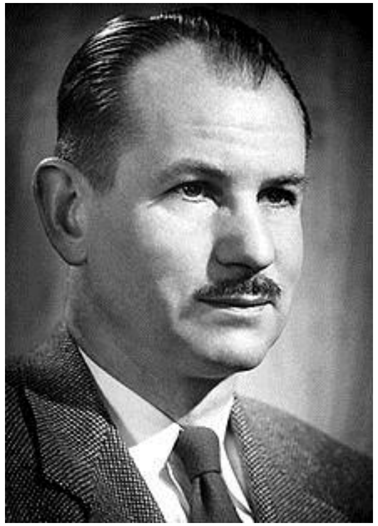
E.Lawrence received the Nobel Prize for this work in **1939** .

Images from https://en.wikipedia.org/

Tools – synchrotron

The synchrotrons keep the accelerated particle on a fixed trajectory by changing the magnetic field synchronously with the increase of particle's energy. The principle was invented by V.I.Veksler in **1944** and independently by E.McMillan in **1945.**

V.I.Veksler
(1907-1966)



E.McMillan
(1907-1991)

Images from https://en.wikipedia.org/

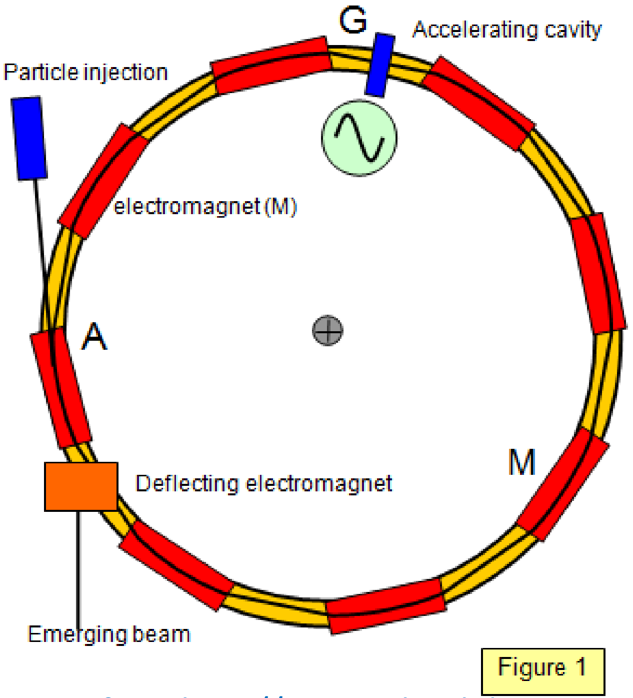


Image from http://www.schoolphysics.co.uk/

Different parts of the particle trajectory in synchrotron serve different purposes: some have dipole magnetic field to curve the trajectory, some – quadrupole field for beam focussing, other contain RF cavities where particles are accelerated etc.

There is an important relation between size of the synchrotron, particle momentum and magnetic field:

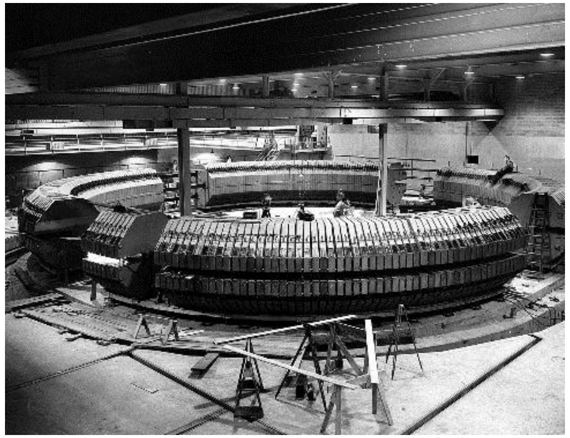
$$r[\mathrm{m}] \cdot B[\mathrm{T}] = 3.33\, p[\mathrm{GeV}]$$

In Large Hadron Collider, **2πr = 27 km**, **2/3** is covered by dipole magnetic field, **B = 8.3 T**. Hence **p = 7 TeV.**

Notice that unlike cyclotrons, synchrotrons cannot accelerate particles from zero kinetic energy. Therefore they need injection and pre-acceleration systems.

Tools – largest proton accelerators

1953

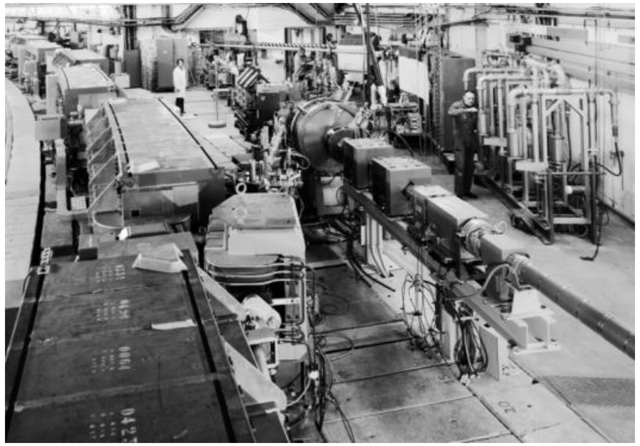


Cosmotron, BNL, USA, **3 GeV**

1957

Synchrophasotron, JINR, USSR, **10 GeV**

1959



Proton synchrotron , CERN, Switzerland, **28 GeV**

1967

U-70, IHEP, USSR, **70 GeV**

1980

Proton supersynchrotron , CERN, Switzerland, **480 GeV**

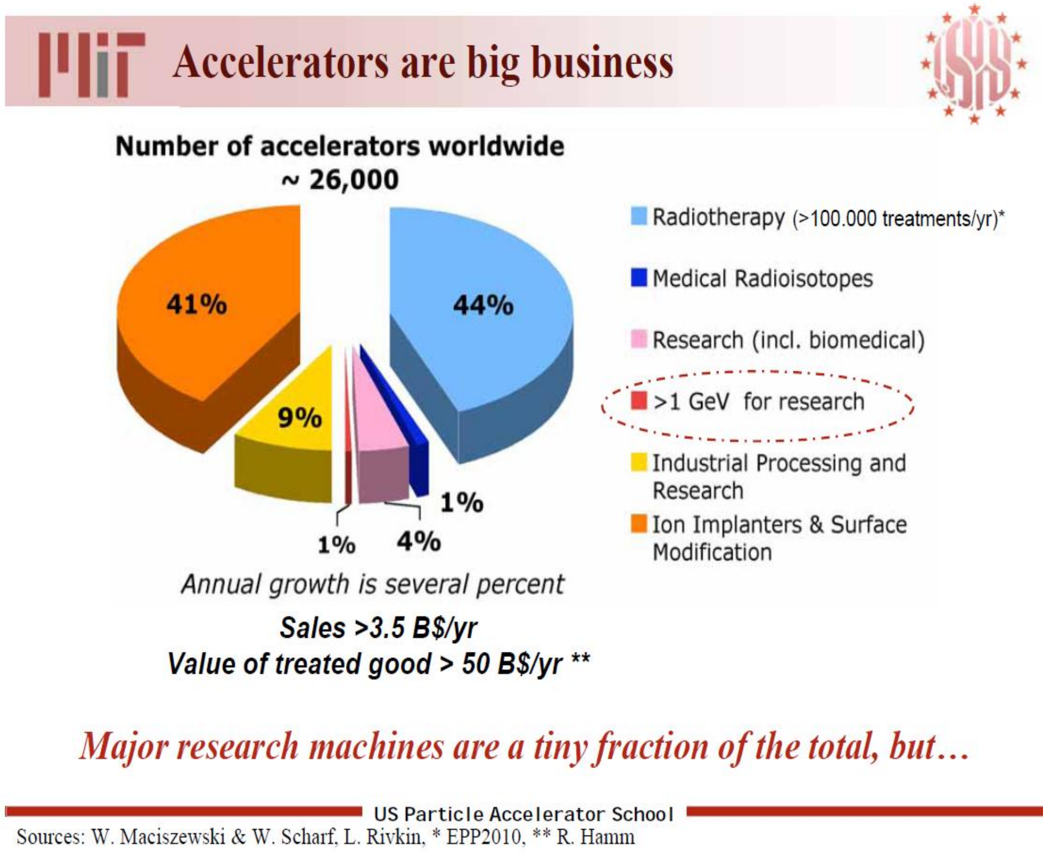
1983

Tevatron, Fermilab, USA, **980 GeV**

2008

Large Hadron Collider, CERN, Switzerland, **7 TeV**

Images from https://en.wikipedia.org/



Accelerators are big business

Number of accelerators worldwide
~ 26,000

- 🟦 Radiotherapy (>100.000 treatments/yr)*
- 🟦 Medical Radioisotopes
- 🟪 Research (incl. biomedical)
- 🟥 >1 GeV for research
- 🟨 Industrial Processing and Research
- 🟧 Ion Implanters & Surface Modification

Annual growth is several percent

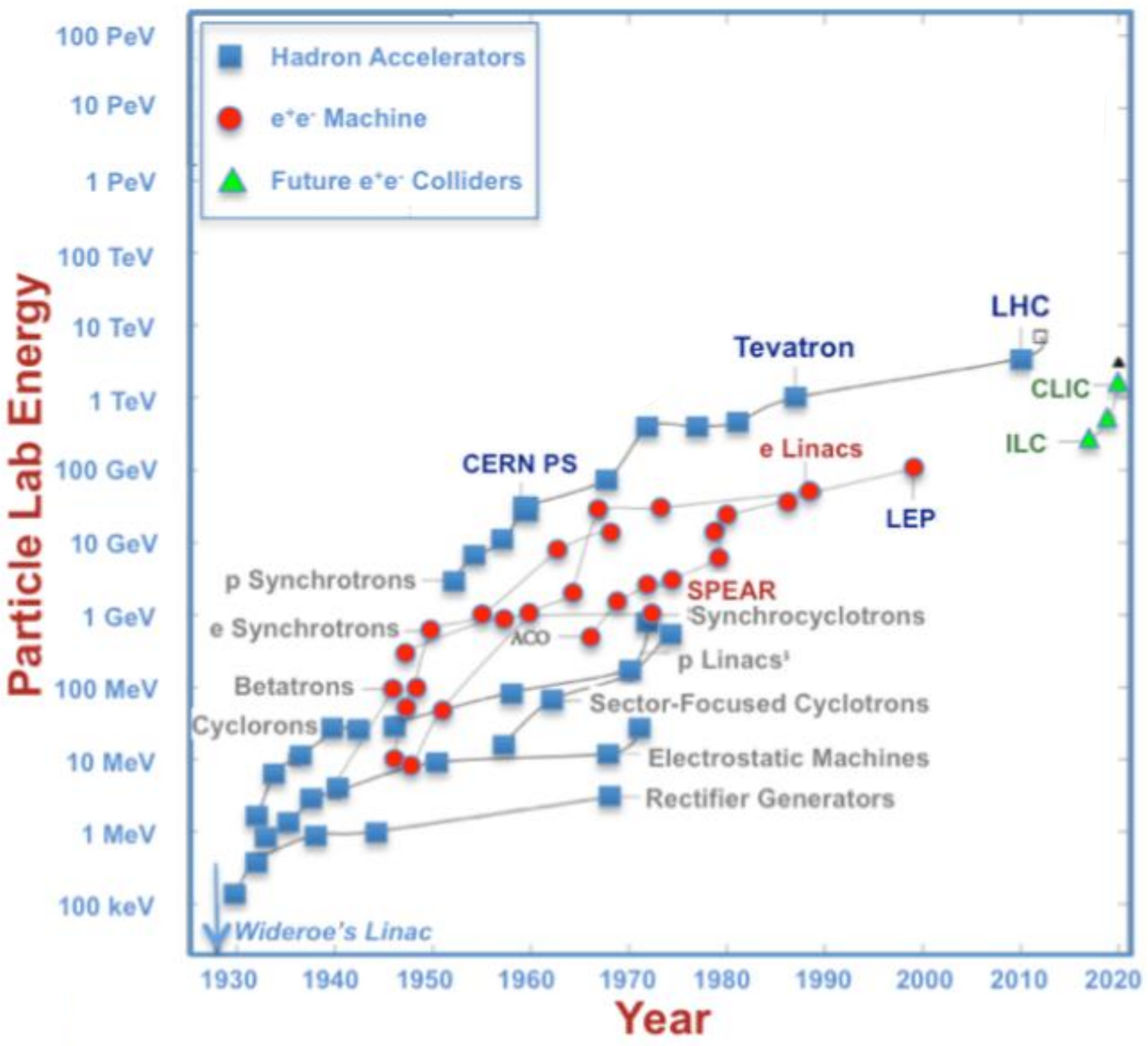
Sales >3.5 B$/yr
Value of treated good > 50 B$/yr **

Major research machines are a tiny fraction of the total, but…

US Particle Accelerator School

Sources: W. Maciszewski & W. Scharf, L. Rivkin, * EPP2010, ** R. Hamm

A famous «Livingston chart» shows the exponential growth of maximal particle energy with time for many decades (it is worth seeing how ideas behind technical design of accelerators appear, rise to maturity, and finally give a place to new, more sophisticated ideas).



A «Livingston chart»

Image from C.E.Pagliarone (2011) J. Phys.: Conf. Ser. 287 012005

Cosmic rays

There is a unique natural source of ultra-relativistic particles – cosmic rays. Some of these particles originate from the Sun, others have extra-solar system origin, including particles from distant galaxies. They were discovered by V.Hess in **1912** (Nobel Prize **1936**) in balloon experiments. The lightest meson – pion and the second generation charged lepton – muon – were first seen in cosmic rays.

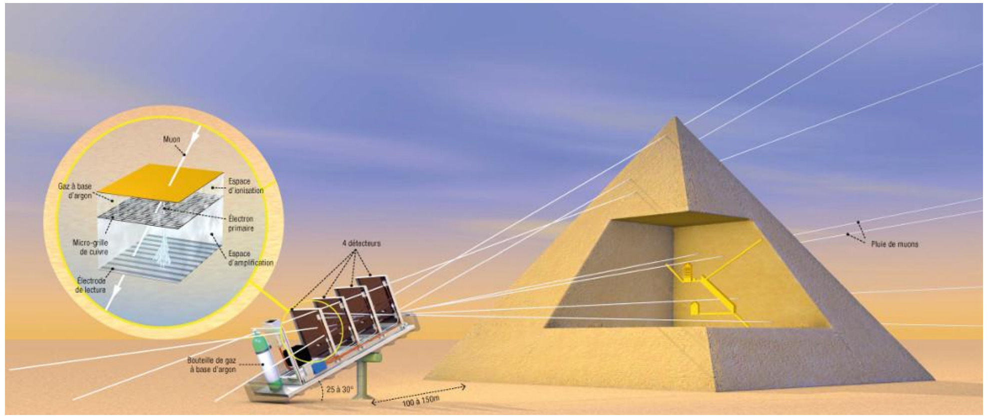
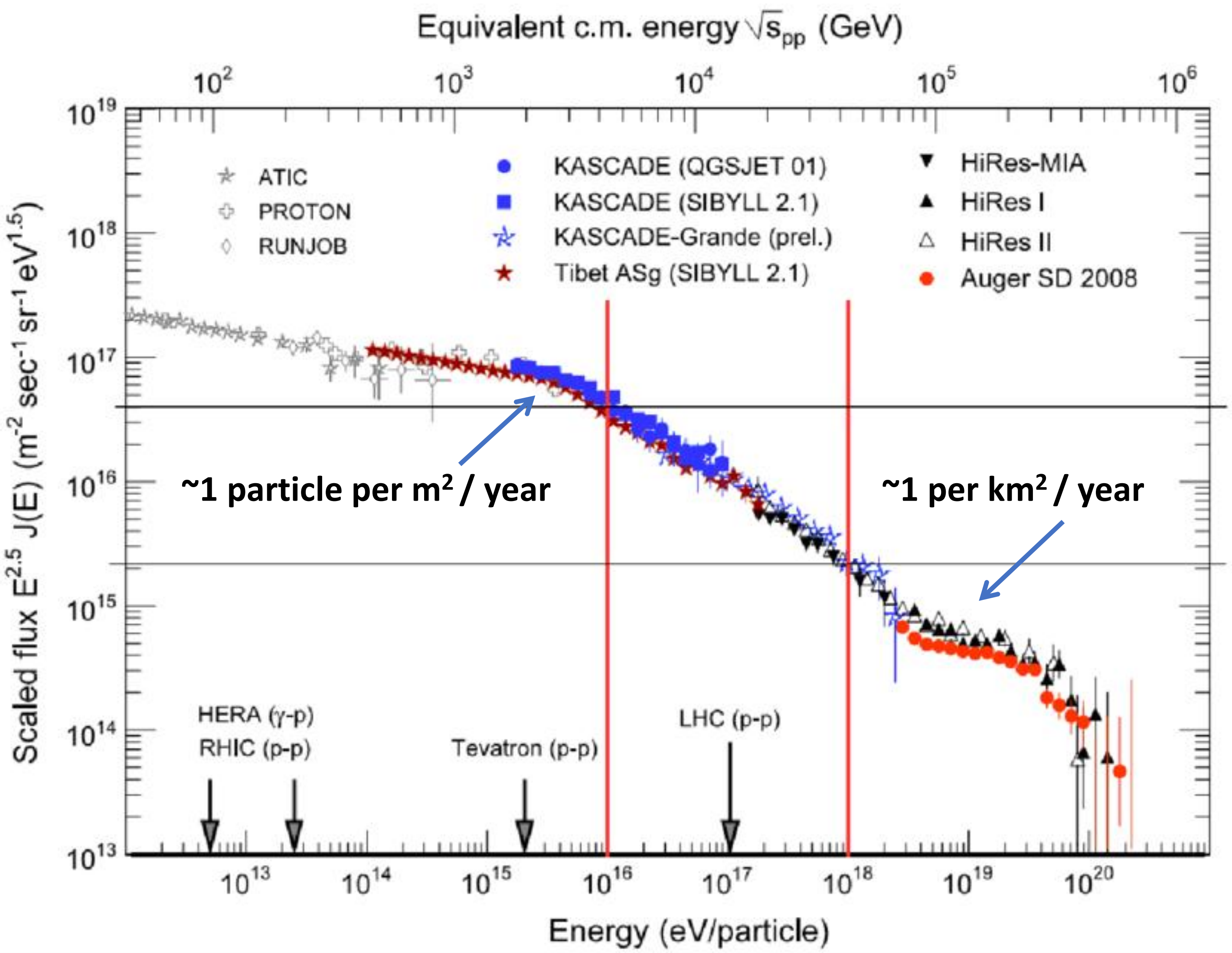


Image from https://www.caen.it/scanpyramids-project/



~1 particle per m² / year

~1 per km² / year

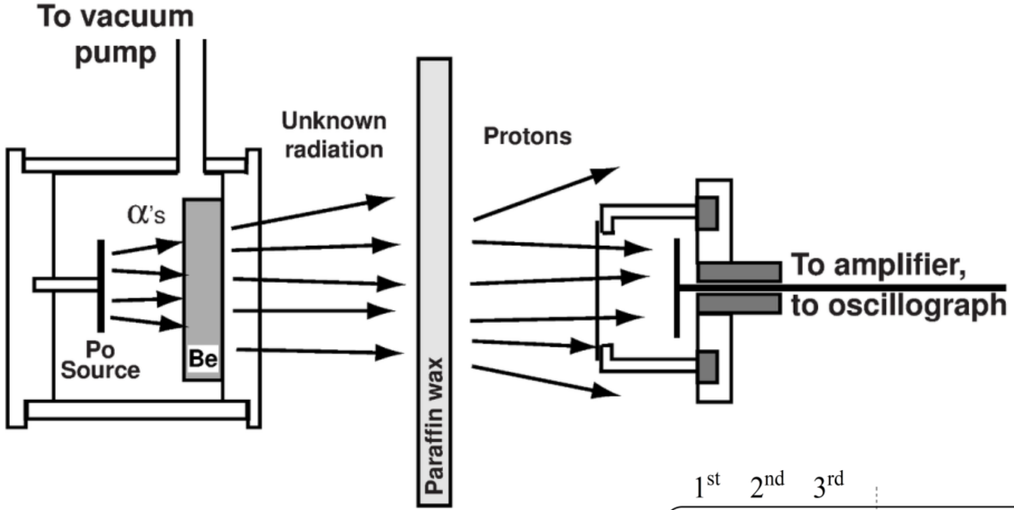
Images from https://en.wikipedia.org/

Unfortunately, too small particle fluxes make it senseless to install standard detectors for studies of collision products

Tools – detectors

Some historical aspects of particle detectors are covered in Parts II and III of this course. Detectors open particle physics era by discoveries of positron (antiparticle of electron, predicted by P.Dirac in **1928**, Nobel Prize **1933**) by C.Anderson in **1932** (Nobel Prize **1936**), neutrons by J.Chadwick in **1932** (Nobel Prize **1935**) and muons by C.Anderson and S.Neddermeyer in **1936**.

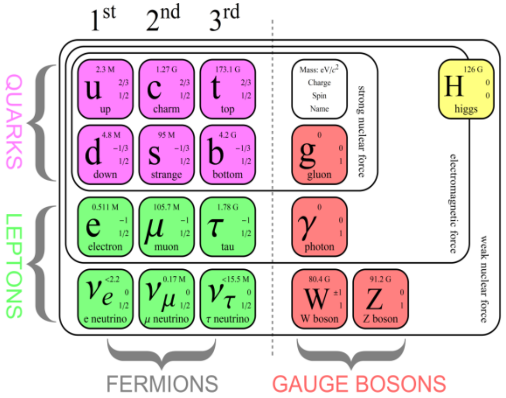
J.Chadwick
(1891-1974)



J.Chadwick's experiment scheme

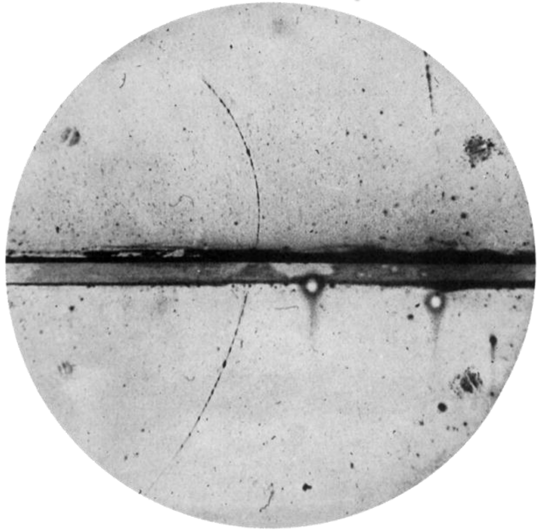
The current stage of this development is summarized in nice picture we started from:

Stay tuned!



C.D.Anderson
(1905-1991)

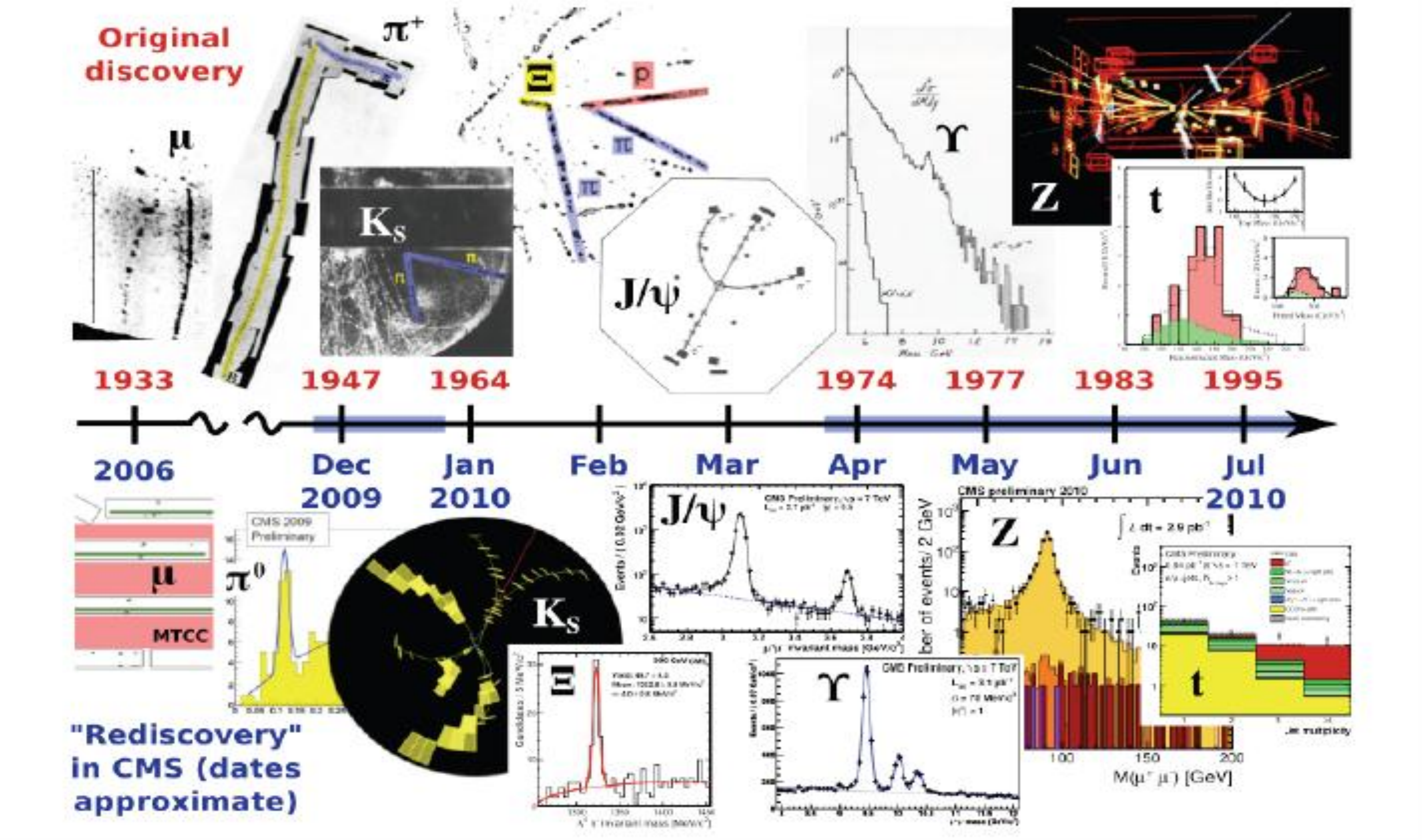
S.Neddermeyer
(1907-1988)



Cloud chamber photograph of the first identified positron.

Images from https://en.wikipedia.org/

Brief History of the Standard Model



Original discovery

1933 1947 1964 1974 1977 1983 1995

2006 Dec 2009 Jan 2010 Feb Mar Apr May Jun Jul 2010

"Rediscovery" in CMS (dates approximate)

Quiz

1. What is the difference between Compton and de Broglie wavelengths?

2. What is the difference between cyclotron and synchrotron?

3. Why does one need not only dipole, but also higher multipole magnets in a synchrotron?

4. What is the largest particle energy observed in the cosmic rays?

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* For ultra-relativistic particles one can consider them as massless, in many cases. Is it possible to accelerate electrons up to **7 TeV** in the Large Hadron Collider tunnel, as it is for protons?

* Calculate the Compton wavelength of a proton with energy **1 TeV**.

Homework problems

1. Protons in Large Hadron Collider are accelerated up to **7 TeV**. What is the difference between the velocity of light and that of the protons in the laboratory frame?

2. The heaviest charged lepton – tau-lepton – has the world average value of the mass **1777 MeV/c^2** and the lifetime **2.9×10^{-13} s**, measured at rest. Assuming tau-lepton produced at Large Hadron Collider has energy **100 GeV**, estimate its travel distance before decay in the laboratory frame.

3. Prove the formula from the slide **33**: $r[\mathrm{m}] \cdot B[\mathrm{T}] = 3.33\, p[\mathrm{GeV}]$

4. Having **c = 299 792 458 m/s** and **ħ = $1.054571817\times10^{-34}$ J·s** in SI units, express the product **ħc** in **GeV·Fm.**